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SOME CORRECTIONS TO R. URBANIAK'S PAPER ON ONTOLOGICAL FUNCTORS OF LEŚNIEWSKI'S ELEMENTARY ONTOLOGY

In the abstract of his recent article (4) Rafał Urbaniak announces:

> We present an algorithm which allows to define any possible sentence-formative functor of Leśniewski's Elementary Ontology (LEO), arguments of which belong to the category of names. Other results are: a recursive method of listing possible functors, a method of indicating the number of possible n-place ontological functors, and a sketch of a proof that Leśniewski's Elementary Ontology is functionally complete with respect to $\{\wedge, \neg, \forall, \varepsilon\}$.

Our claim is the author presented neither a correct algorithm, nor a correct method for intended tasks the sketch being just sketchy and therefore hard to judge. Still, if we were to base the proof on the faulty results we could obtain wrong conclusions.

We omitt much of the author's explanations, "definitions" (which sometimes prove to be schemes of definitions and sometimes are not very clear or exact) etc.[1] concerning various languages for ontology and so on, which do not always seem to give much help in proving the main theses. So, informal style might prove more efficient in answer to informal declaration of intentions of the author in the discussed paper.

The author writes in some places he wants to define functors of Elemen-

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I thankfully remember Prof. Ludwik Borkowski who introduced me to the systems of Leśniewski and thank Dr Stanisław Majdański who taught me semiotics.

[1]On p. 28 the undefined symbol $\bigcap$ appears which might have not be very hard to accept as a sign of some kind of product were it not at the critical point of generalising from two argument functors (and semantical stati or "statuses") to many-argument ones. Another example are schemes of definitions called simply "definitions" as in the case of those marked as 3.7, 4.2, 4.5. Again, $f_1, ..., f_k$ is described on p. 16 as a set of logical constants and on the same page below $f, f_1, ..., f_n$ are described as variables which "represent functions". Occasionally, R. Urbaniak himself admitts some ambiguity or imprecision. See p. 16 footnote 2, p. 25, p. 34 footnote 21, p. 35 footnote 22 etc.

tary Ontology[2] but he actually means sentence-formative functors of name arguments. However, even the notion of something definable or definiability is not selfevident.[3] It is said the definitions are in language of ontology "following the style of Leśniewski" (though not preceded by universal quantifier(s)) but how to understand the claim that **all** functors are definiable with the use of some other? The rules for admittance of definitions were given by Leśniewski and it is more or less known which functors are allowed or definable. So, it is no major problem. Rather, it might be meant that within the characterisation of $\frac{s}{n,\dots,n}$ functors by a chosen method all of obtained differentiations of these functors are expressed with the definitions of Elementary Ontology. The functors are beeing described in metalanguage via concept of "semantical status" (or SeS). In the Definition 4.7 (p. 31) of $SOFk_{n\geq 3}$ for functors $\delta_{k\geq 3}$ the expression $\sigma^1_{n\geq 3}\langle\pi_1,\dots,\pi_{n\geq 3}\rangle$ is being used in the definiens. Unfortunately, this expression has not yet been defined. Supposedly, it is to be understood intuitively per analogiam ad "2-Place Ontological Functors" defined earlier. [4] However, even this definition is not easy to accept for it is not very precise.[5] Therefore, although one can understand intentions of the author it would be more difficult to discuss

[2]He says: "Those are *Ontological Functors* (OF). They are those specific functors which distinguish *Elementary Ontology* from *Prototetics* [sic]. As it is obvious, these functors are of syntactical category $\frac{s}{n_1,\dots,n_c}$". See p. 24. (Cp. also p. 16 where it is written : "The purpose of this paper is to provide an effective method of defining any [sic!] ontological functor".) However, the adopted terminology is somewhat misleading as the commented part of Ontology does not contain expressions formed with the name-forming functors although Elementary Ontology was described (p. 15) as just such a part of Ontology "in which the only variables are name-variables". Incidently, the notation for the category of expressions, $\frac{s}{n_1,\dots,n_c}$, is a bit strange, for the same category n seems to be marked with different signs: $n_1,\dots,n_c$. Seometimes both styles (with or without indexes) are used on the same page e.g. see p. 24.

[3]Despite the fact that the author already in the second sentence of the article says: "A system is *functionally complete if and only if* all its possible functors may be defined with the use of 1-, and 2- place its functors as the only functors."

[4]See: "The **Definition 4.2.** Functor δ_1 is a POF_1 *if and only if* $\exists_{1\leq i\neq j\neq\dots\leq 3}[\delta_1\langle\pi\rangle \equiv \underbrace{SeS_1\langle\pi\rangle = I.i. \vee \dots \vee Ses_1\langle\pi\rangle = I.j.}_{\text{the number of disjuncts is 2 or 3}}]$".

[5]In the first argument of the equivalence "$\delta_1\langle\pi\rangle \equiv SeS_1\langle\pi\rangle = I.i.$", π is probably meant to refer to objects and in the second one to names. R. Urbaniak writes this way throughout his paper. It all starts with the "$Ext(\pi)$ understood as "extension of π" (p. 18) and the definition 3.1 of "Semantic Status" where the expression "a given name π" occurs despite the initial (p.16) declaration that "$\pi,\pi_1,\dots,\pi_n$ represent names".

them in the original formal presentation.

While proceeding in "Leśniewski style", R. Urbaniak starts by defining the simplest sentence-forming functors of name arguments. "OF-Definitions 1-3" (of some "basic" functors of $\frac{s}{n}$ category) use ε which was not defined earlier. He gave a set theory explanation of ε earlier but did not choose any of the axioms for Ontology to clarify the meaning of the constant from within the system of Ontology. Of course, one might have some intuitive meaning of set theory but let us remind that the concepts of that theory might be not better than those of Leśniewski's Ontology (LO). Besides, if the author wants to show all functors of LO are definable by means of ε as the only functor of LO he should give the meaning of ε at least via axiom(s) and rules of the system. Not to mention some terminological explanations or given way of reading the expressions. Otherwise, the definability depends on the correct characterization (in prospect) of the meaning of ε. Which might be somewhat doubtful a task in view of the richness of the Leśniewski's (even Elementary) Ontology system(s) on one hand and simplicity of its, alternatively chosen, axioms on the other. Probably, the axioms and rules are taken for granted. Somewhat surprising, however, is the fact that even in the semantical deliberations R. Urbaniak omitted the characterisation of ε through "semantical status".

As for semantical explication of the meaning of functors, R. Urbaniak starts with a presentation of the Czesław Lejewski's "Ontological Table"[6] which displays, aspectively, any possible semantic status of both single names and pairs of names. The status is described both by the character of the names in question (if they are empty, single or "shared") and, in the case of the pairs of names, by the presence or not of their nonempty common extention and of a nonempty part of the extention of one of them laying outside of the extention of the other. Very much like "Euler circles" enriched with the marks for one of the three kinds of the names: empty, single or shared. He explains:

> Knowing the SeS_2 of any given pair of names, we not only know, what the SeS_1 of each of these names is, but also, in what relation to each other remain the sets of their designates.

He also says:

[6] See (1).

> Quite helpful in defining $SeS_{n\geq 3}$, say σ^1, seems to be the fact, that if we have $\sigma^1_{n\geq 3}\langle \pi_1, ..., \pi_{n\geq 3}\rangle$, we know all SeS_2 of pairs from $\{\pi_1, ..., \pi_{n\geq 3}\} \times \{\pi_1, ..., \pi_{n\geq 3}\}$ i.e. from $\{\pi_1, ..., \pi_{n\geq 3}\}^2$. Also, it seems [sic!] to work in the other direction: if we know all SeS_2 of all pairs from $\{\pi_1, ..., \pi_{n\geq 3}\}^2$, we know what the $\sigma^1_{n\geq 3}\langle \pi_1, ..., \pi_{n\geq 3}\rangle$ is. [...] In other words, a SeS is a SeS of more than two names *if and only if* it is identical with intersection of SeS_2-es of all pairs of names which were taken under consideration.

However, one cannot precise sufficiently the relation of extentions of more then two names just by conjunctions of relations of pairs of names. For example, let us say we know some people in a room are energetic and young, some are energetic and wise and some are young and wise. Still, we do not know if there are any people in the room who are not only energetic, but also young and wise! Let alone how many of them possess all of those three qualities: many, one or none.

Therefore, we need some other procedure to describe functors as characterised both by the relations of the intersections of extentions of names, nominal arguments of the functors, (and their complements) and differentiation of the qualities of any such a functorial extention according to the "cardinality" of the extentions of the names. One of the correct procedures seems to be as follows. First, one need to define all posible "intersections" of extentions of k names. The names are meant to be arguments of the projected functor. There are $2^k - 1$ such intersections. (To be precise, they are intersections of the extentions of the names or of the complements of those extentions minus one case i.e. that of the intersection of the complements of the extentions of all names under consideration.)[7] Then, any such intersection may be empty, single or shared. So there are 3^{2^k-1} possibilities of basic states of such nature. They could be represented as sequences of $2^k - 1$ elements each of which would be marked as either empty, single or shared.

Let us notice they do not correspond to C. Lejewski's table and R.Urbaniak's SeS-es for they are characterised by the description of number (none, one or many) of the designates from any intersection of the extensions of

[7] If we allowed for the intersection of all the complements we would be even closer to the Venn method of describing extentional relations of sets.

the names whereas C. Lejewski's characterisation mostly omitted the description of the nonempty part of an extension which reaches beyond some other extention or of a the part of a extention which is shared by two extentions of some "shared" names of many designates.

One might argue in favour of our method by pointing to the fact that it is very easy to count different possibilities offered by our means. Also, when we are interested if names are empty, single or shared why should not we be interested in the same status of the intersections of their extentions and their complements? Besides, by describing basic semantic status our way we get more precise description than one presented by R. Urbaniak. Then, we may give separate names to all different sets of these basic semantic states. They would correspond to functors of the category $\frac{s}{n, \ldots, n}$ (of sentence-forming functors of nominal arguments)[8] which form a thesis if the extentions of names and their complements have the described qualities. Any relation of names described by Lejewski-Urbaniak's method would have its equivalent in our collection.

For example, in Lejewski's table we have as basic semantical status II.10 where names a and b are "shared" and their extentions are nonequal and the extention of a is included in *b's.* In our approach this is equivalent to **two** basic situations characterised additionally: the extention of the intersection of b and of complement of the extention of a contains either 1) one, or 2) many elements. These we treat as different basic situations.

We leave aside the question of how to describe many-argument functors exactly in the style of C. Lejewski. The result does not seem to be very important neither philosophically nor mathematically. Let us just notice we may characterise the "cardinality" (the number of elements) of any chosen extention in very different ways by choosing not just three characteristics ("none", "one", "many") but much more. Indeed, Lejewski's idea to choose and join some characteristics which exclude each other and exhaust all possibilities to characterise the extention of a name and, in the case of two names, both their cardinality and inclusion or intersection of their extentions[9] may help in clarifying some, especially basic, differences between functors of name arguments. Consequently, different sets

[8] In the original text: $\frac{s}{n_1, \ldots, n_c}$.

[9] We would prefere to consider all posible intersections of extentions of names or their complements (the intersection of complements of the extentions of all the names could be, optionally, omitted) alongside their cardinality.

of numbers of designates of intersections of extentions of names and their complements could be used to characterize all sentence-forming functors of name arguments. In the semantical considerations one could even think of the characteristic called "infinity".

Finally, let us very briefly comment on a note by Urbaniak concerning nominalism:

> Leśniewski, in fact, was a nominalist. Hence, sometimes, his systems [...] are believed to be nominalistic. However, the deliberations hitherto led show us only that what suffices as a model of given LEO-langague, is a set of objects.

In answer to this, let us note it seems the general tendency of Leśniewski was one towards simplicity of principles. This is probably why although he praised Aristotle he sought of a nominalistic solutions to the problem of language. Let us simplify: instead of talking of sets and their elements it is simpler to speak of names and their designates or, having already formed a correct language, simply about things (well, of various categories).[10] Also, instead of speaking that a sentence is true it is simpler to say just the true sentence (although some problems remain e.g. the explanation of quantifiers). Maybe, this kind of tendency towards simplicity is partly to blame for differences of approach towards logic, especially semantics, between Leśniewski and Tarski. Anyway, the language of Ontology and its concepts have advantages over those of set theories in many respects.[11] Therefore, the remark about how a set of objects "suffices as a model" sounds like some misunderstanding of efforts of Leśniewski. Perhaps, a more philosophical approach could show more advantages of the Stanisław Leśniewski's way.

References

[1] C. Lejewski, *On Leśniewski's Ontology*, Ratio **V**, no. 2 (December 1958), pp. 150–176.

[10]Or of object(s) π instead of $Ext(\pi)$ - the expression which was introduced on p. 18 after some allusions to a relatively understandably described "*function of extension of names*". Nonetheless, the description presupposes the concept of function and other notions of the set theory. Leśniewski could have said: "I don't understand." Other logicians too were critical about concepts of set theory, e.g. Thoralf Skolem (2).

[11]Cp. footnote 60 in §4 of (3).

[2] T. Skolem, *Einige Bemerkungen zur axiomatischen Begründung der Mengenlehre*, Matematikerkongressen i Helsingfors den 4-7 Juli 1922, [in:] Den femte skandinaviska matematikerkongressen. Redogörelse, Helsinki 1923.

[3] A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych (The concept of Truth in the Language of the Deductive Sciences)*, Prace Towarzystwa Naukowego Warszawskiego, Wydział III No. 34, Warszawa 1933.

[4] R. Urbaniak, *On Ontological Functors of Leśniewski's Elementary Ontology*, Reports on Mathematical Logic **40** (2006), pp. 15–43.

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RESPONSE TO A CRITIC (DEFINABILITY AND ONTOLOGY)

I would like to thank Mr Borowski for his comments, I appreciate his time and effort. It is always uplifting to learn that a topic which one has considered quite hermetic can stir up such emotions. I'll just briefly respond in a rather relaxed manner to what I think the main points raised by Mr Borowski are.

1. The first complaint is that in definitions (which Mr Borowski refers to with scare quotes: "definitions") I sometimes use schemes rather than formulas. Well, what can I say? I do. For instance, when I define the intersection operator for semantic states, I give a schema that accounts for intersections two-, three-, four-, and so on, -place semantic states. But I'm pretty clear about it: I use meta-variables introduced on p. 16 (the very first page of the paper), and (rather unsurprisingly) formulas with meta-variables are schemas. I don't

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think there is a serious risk of ambiguity here. One reader's "sloppy" is another reader's "saving fifteen minutes of my life".

2. In the similar vein, in footnote 1 of his paper, Mr Borowski says that an undefined symbol ($\bigcap$) occurs on p. 28 of my paper and he complaints about me requiring the reader to guess what the symbol means. Well, yes, in the spot that Mr Borowski refers to the symbol occurs undefined indeed, but it is a very straightforward generalization of the binary intersection (in fact, only finite generalized intersections are being considered, so there's no worry about the divide between finite and infinite cases).

3. Mr Borowski observes that I sometimes speak as if I wanted to define functors of Elementary Ontology, whereas what I do, is I discuss definability of sentence-formative functors of Elementary Ontology. Strictly speaking, I said: "[Ontological Functors] are those specific functors which distinguish Elementary Ontology from Protothetics. As it is obvious, these functors are of syntactical category $\frac{s}{n_1,\ldots,n_c}$." (p. 24) Indeed, Elementary Ontology contains also name-forming functors, but still, to my mind, the most important step from Protothetic to Ontology consists in introducing name variables and sentence-forming connectives that take name variables as arguments. This, of course, was just an informal claim (and Mr Borowski has a right to believe that name-forming functors are equally important). Anyway, I made it clear that the functors I will refer to as 'ontological functors' will be sentence-formative.

4. In footnote 2 Mr Borowski worries that at some point I used numerical subscripts for syntactic category indicators. That's a typo, sure, and Mr Borowski is quite correct in pointing it out. However, since it's a fairly straightforward one to figure out, I hope to be able do my best and to go on with my life without shooting myself because of it.

5. Mr Borowski remarks:

> However, even the notion of something definable or definiability is not selfevident.[1] It is said the definitions are

[1] I presume he meant: 'even the notion of something *being* definable [or *definability*] is not self-evident.'

> in language of ontology [...] but how to understand the claim that all functors are definiable[2] with the use of some other? The rules for admittance of definitions were given by Leśniewski and it is more or less known which functors are allowed or definable. So, it is no major problem.

This is pretty much like saying that given certain obvious syntactic restrictions on definitions of sentential connectives in a classical extensional propositional language based on negation on conjunction, it doesn't make sense to give a functional completeness proof because it is already known what connectives are definable. Mr Borowski seems to conflate the semantic and the syntactic notions of functors. A typical functional completeness result draws a connection between these two: it says that given a certain primitive vocabulary, certain syntactic restrictions on what definitions look like, and a certain semantics, all semantically relevant functions can be defined. In the case of

Elementary Ontology, the connection to be drawn is between functors that are syntactically definable and semantically defined functors, not between syntactically definable functors and between syntactically definable functors themselves (that would be quite boring and rather obvious).

6. Mr Borowski complained that I use "a set theory explanation of ε" (I assume he meant 'a set-theoretic explanation') without choosing "any of the axioms for Ontology to clarify the meaning of the constant from within the system of Ontology".

   Sure, I use set theory. But if Mr Borowski believes that using set-theory is a serious objection against semantical considerations, then even if he restricts himself to those papers that use set theory which are about Leśniewski's systems, he probably should be writing one critical review a week for the next couple of years to assure the purity of Leśniewskian studies. Of course, there is a chance most of mathematicians and logicians who use set theory are insane and waste their lives working within theories that don't make sense.[3] But even

[2] I presume he meant: *definable.*

[3] By the way, it's rather known that the axioms of Ontology are not really good at determining the meaning of the epsilon operator (1; 2).

if that's the case, there seems to be nothing immediately wrong in playing along and using what they believe makes sense to explain to them some other things.

7. Another remark is:

> If the author wants to show all functors of LO are definable by means of ε as the only functor of LO he should give the meaning of ε at least via axioms and rules of the system.

In what Mr Borowski wrote, I really couldn't find an argument for the claim that I really should do that. For instance, functional completeness proofs for a classical propositional language can be given without giving any proof system for classical propositional logic. What needs to be given is a semantics and a syntactic account of what a definition looks like. I gave a semantics (I know, it was an evil semantics because it used set-theory; what can I say, if using set theory makes one evil, I'm evil).

8. In footnote 5, Mr Borowski says that "in the first argument of the equivalence $11\delta\langle\pi\rangle \equiv SeS_1\langle p\rangle = I.i'$" π is probably meant to refer to objects and in the second one to names." I really have no idea why he thinks that.

9. Actually, Mr Borowski raises one rather valid point: he says that "one cannot precise sufficiently the relation of extentions [I take it, he meant 'extensions'] of more then [I take it, he meant 'than'] two names just by conjunctions of relations of pairs of names." I agree that his counterexample works against the way I defined the construction in question. Rather, I should have admitted a few basic boolean name-forming operators on names as primitive (=defined only in terms of semantics) and defined extensions of more names in terms of the relations between the extensions of pairs of names and pairs of certain (finite in number, but exhausted methodically) boolean combinations of those names. The required changes are rather straightforward, so I won't bother explicating them.

10. Mr Borowski suggests that "instead of speaking that a sentence is true it is simpler to say just the true sentence". Sure. If you can

actually display it. One of the main functions of truth-predicate is to allow us to state truth of statements we can't display or to express quantification over sentences that we are either unable to display or over sentences that are infinite in number and thus for fairly obvious practical reasons cannot be listed. This issue is rather unrelated to what the original paper was about, though, and I'm not sure why Mr Borowski brings it up.

11. In the similar vein, near to the end of the paper Mr Borowski complaints that I use the notion of function and

    Leśniewski could have said "I don't understand".

    Indeed, Leśniewski could have said many things. If he were alive. But he's not. Of course, Leśniewskian scholars are free to lock themselves in their ivory towers of allegedly pure Leśniewskian systems, murmuring "I don't understand" whenever a concept that employs set-theory is used. But I tend to suspect that this is not the best strategy if one wants Leśniewski's systems to play any interesting role in current research in logic.

Once again, I would like to thank to Mr Borowski for his criticism – he pointed out a mistake in one of the definitions, and the review brought up to my attention the need for extreme clarity which I, over-relying on the reader's common-sense, might have neglected.

References

[1] R. Urbaniak, *Some non-standard interpretations of the axiomatic basis of Leśniewski's Ontology*, The Australasian Journal of Logic **4** (2006), pp. 13–46.

[2] R. Urbaniak, *A note on identity and higher-order quantification*, The Australasian Journal of Logic **7** (2009), pp. 48–55.

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