In memory of Wim Blok

This *Special Issue* of *Reports on Mathematical Logic* is dedicated to the memory of the Dutch logician WILLEM JOHANNES BLOK, who died in an accident on November 30, 2003. This was an immense loss for the academic community of logicians and universal algebraists, and more particularly for *Reports on Mathematical Logic*.

WIM, as everyone called him, published a number of papers in *Reports on Mathematical Logic*, was a member of our *Editorial Board* for several years, and was one of the guest editors of the volume [1]. His own research work, taken collectively, corresponds closely to the fields most often represented in the journal: algebraic methods in modal and substructural logics, abstract algebraic logic, algebraic properties with logical significance in varieties and quasi-varietes, finite model properties, etc.[1] We believe that the papers we have gathered for this *Special Issue* pay fitting tribute to WIM's work, and in particular they exemplify the seminal character of some of his most original ideas.

Among these ideas we find the theory of *algebraizable logics* and the classification scheme of deductive systems known as *the Leibniz hierarchy*, which he introduced and studied with DON PIGOZZI in [3, 4, 5] and which were later expanded on mainly by CZELAKOWSKI, HERRMANN, JANSANA and RAFTERY[2]. One of the most profound influences of these new ideas was that they stimulated global thinking in algebraic logic; that is, besides working on the particular algebraic semantics of a particular logic it is now possible to work on general properties that concern classes of logics taken as a whole[3]. We think that all the papers included in this *Special Issue* share this point of view.

The key idea in [3] is to describe algebraization by a pair of two mutually inverse faithful and definable translations between the object logic and a quasi-equational logic relative to a class of algebras. The object logic was

[1]The short but very pertinent obituary note [14] by JAMES RAFTERY contains a more detailed, systematic analysis of WIM's contributions and their impact.

[2]The survey papers [10, 12] and the monograph [6] contain more detailed accounts of the evolution of the subject and of each author's contributions, together with the necessary references.

[3]One can consider RASIOWA's celebrated monograph [15] and the general theory of logical matrices, cf. [18], as precursors of this trend.

originally a deductive system of propositional logic[4], but it quickly became clear to many that it was possible, and natural, to extend this idea in several directions and at different levels of generality. Two of the papers in this *Special Issue* belong to this trend.

GIL and REBAGLIATO's paper *Finitely equivalential Gentzen systems associated with arbitrary finite algebras* continues these authors' earlier work [13] on extending the ideas of algebraizability and of the Leibniz hierarchy to Gentzen systems. Here, they show that an arbitrary finite algebra always defines a finitely equivalential Gentzen system in a uniform way. They establish some general properties of this kind of systems, and show that the algebraic counterpart of the said Gentzen system coincides with the quasivariety generated by the algebra.

VOUTSADAKIS started in [16] the study of the same idea, and more generally that of equivalence of deductive systems, in the framework of π-institutions, a very general and abstract notion of deductive system rooted in category-theoretic notions associated with abstract model theory. Later he introduced other notions of abstract algebraic logic into this framework, such as the theory of full models [11]. In the present paper *Categorical abstract algebraic logic: Full models, Frege systems, and metalogical properties* he formulates an institutional analogue of the property of congruence and analyses how it helps in the preservation of other metalogical properties such as conjunction, disjunction, the deduction-detachment theorem, and two versions of reductio ad absurdum.

In partial contrast, RAFTERY's paper *The equational definability of truth predicates* explores a new direction in the study of the Leibniz hierarchy without leaving its original framework, that is, the algebraic study of deductive systems of sentential character. Most of WIM's work in abstract algebraic logic concerned the largest class in the hierarchy, namely the class of *protoalgebraic logics*, which he and DON introduced in [2]. One of the several different ways in which this class is characterized is by the property that the Lebniz operator is monotonic on the lattice of theories of the logic. Initially it was believed that this class encompassed all logics whose matrix semantics was amenable to the lattice-theoretic and model-theoretic methods of traditional algebraic logic and of the more modern universal-

[4] Already in Appendix C of [3] the authors show how to apply the method to account for the algebraization of first-order logic in terms of cylindric algebras.

algebraic techniques[5]. However, later on, other logics arose which were non-protoalgebraic but could nevertheless be treated algebraically[6]. In his paper RAFTERY studies several issues of the general theory, in particular that of the equational definition of the truth condition of a matrix semantics, without the assumption of protoalgebraicity. The main tools in his study are the Suszko operator, introduced by CZELAKOWSKI in [7], and Suszko-reduced matrices; these play, to a certain extent, a role similar to that played by the Lebniz operator and reduced matrices for protoalgebraic logics. The relevant properties of both operators are here injectivity and complete order reflection.

Consideration of the Leibniz hierarchy naturally led other people to think about other classification schemes that also have an algebraic significance. One of these revolves around the replacement properties the logic can have, and originates what is termed the *Frege hierarchy* [8, 10]. Two papers in this *Special Issue* deal with the lowest level of this hierarchy, the class of *selfextensional logics* introduced in [17]. There WÓJCICKI characterized selfextensional logics as those admitting a referential semantics, an abstract version of the general frame semantics of many intensional logics. A connection with WIM's work can also be found here, recalling his important contributions to the field of modal logic.

In their paper *Referential semantics: duality and applications*, JANSANA and PALMIGIANO establish a duality between certain categories of atlases (DUNN's version of generalized matrices) and of referential algebras. This duality arises as an abstraction of the well-known dualities between categories of algebras and of general Kripke frames for several propositional logics, and holds uniformly for every selfextensional logic. Furthermore, they use this duality to obtain a characterization of fully selfextensional logics, another of the classes in the Frege hierarchy, introduced in [11].

The paper *On truth schemes for intensional logics* by CZELAKOWSKI and DZIOBIAK is a first step in what appears to be a new, far-reaching research programme, that of developing a general analysis of relational

[5] The authors of [2] say that "*[the class of protoalgebraic logics] includes all the (non-pathological) propositional logics considered in the literature we are aware of*" (page 338).

[6] The first of these, and a prototypical example due to its simplicity, is the conjunction-disjunction fragment of classical logic. Other examples are the implication-less fragment of intuitionistic logic, BELNAP's four-valued logic, several subintuitionistic logics and several positive modal logics. See RAFTERY's paper for more examples and references.

semantics at a higher level. These authors use a second order language to express conditions appearing in the definition of the truth of a formula at a point in a relational model. The paper focuses on canonical models, and shows that if a logic is selfextensional then it is always possible to formalize these conditions in order to define a canonical frame adequate for the logic.

The paper with the greatest typically algebraic content in the *Special Issue* is *Almost minimal varieties related to fuzzy logic*, by KATOH, KOWALSKI and UEDA. This paper is connected to WIM's work in two ways. On the one hand, it studies several classes of strongly algebraizable logics by means of the associated varieties, and it does so from a global perspective rather than by isolating each logic and its variety. On the other hand, the logics under study belong to the group of *substructural logics* [9], a field of growing interest and one to which WIM made important contributions in the last part of his life (some of them have been published only posthumously).

We believe that these papers constitute a much deserved, though modest homage to WIM's personality and influence in shaping algebraic logic in the last quarter of the twentieth century. We warmly thank the authors who have contributed and hope that the readers will enjoy and appreciate these papers.

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