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ON ONTOLOGICAL FUNCTORS OF LEŚNIEWSKI'S ELEMENTARY ONTOLOGY

A b s t r a c t. We present an algorithm which allows to define any possible sentence-formative functor of Leśniewski's Elementary Ontology (LEO), arguments of which belong to the category of names. Other results are: a recursive method of listing possible functors, a method of indicating the number of possible n-place ontological functors, and a sketch of a proof that LEO is functionally complete with respect to $\{\wedge, \neg, \forall, \varepsilon\}$.

1. Introduction

By *Elementary Ontology* (LEO) we mean this part of Leśniewski's Ontology, in which the only variables are name-variables (and hence, quantifiers quantify only name-variables).

A system is *functionally complete if and only if* all its possible functors may be defined with the use of 1-, and 2- place its functors as the only functors. By 'System S is *functionally complete with respect to the set*

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of logical constants $\{f_1, \ldots, f_k\}$.' we mean that all its possible functors may be defined with the use of $\{f_1, \ldots, f_k\}$ as the only logical constants. By 'System S is *F-functionally complete with respect to the set of logical constants* $\{f_1, \ldots, f_k\}$' we mean that all the system's functors from the set of functors F may be defined with the use of elements of $\{f_1, \ldots, f_k\}$ as the only logical constants.

The purpose of this paper is to provide an effective method of defining any ontological functor (This notion will be explicated later on. Tentatively - those are the functors specific to LEO.) of LEO with the use of $\{\wedge, \neg, \forall, \varepsilon\}$ only, and thus to sketch a proof for the thesis that LEO is functionally OF-complete with respect to $\{\wedge, \neg, \forall, \varepsilon\}$ (where OF denotes here the set of ontological functors).

2. Basic Notions

We omit definitions of $\exists$, $\vee$, $\rightarrow$, $\equiv$ by means of $\{\wedge, \neg, \forall\}$ as obvious.

Let $\{a, b, a_1, \ldots, a_n b_1, \ldots, b_n\}$ be a set of name variables of Ontology. Next, we apply the following convention for variables in meta-language: $\varphi, \psi, \varphi_1, \psi_1, \ldots, \varphi_n, \psi_n$ represent sentences; $\chi, \chi_1, \ldots, \chi_n$ represent sentential-expressions (including sentences and sentential formulas); $\tau, \tau_1, \ldots \tau_n$ represent sentential formulas; $\mu, \nu, \mu_1, \nu_1, \ldots, \mu_n, \nu_n$ represent name-variables; $\pi, \pi_1, \ldots \pi_n$ represent names; $\alpha, \alpha_1, \ldots, \alpha_n$ represent name-expressions (including names and name-formulas). Let $\sigma^1, \ldots, \sigma^n$ be variables[1] each of which can be substituted only by a name of a Semantic Status (the notion will be defined later on).[2] Let also $\delta, \delta^1, \ldots, \delta^n$ represent functors of category $\frac{s}{n_{1,\ldots,n_k}}$.[3] Let $L, L_1, \ldots, L_n$ represent languages. The variables $f, f_1, \ldots, f_n$ represent functions.

We distinguish: *Unshared* names, each of which names exactly one object (e.g. 'Socrates'). According to the long-lasting tradition, we accept the view that names signify without time. It means that if a name names an

[1]In the case of this variables they are differentiated by upper case numbers, since the lower case number informs us about the number of names the Semantic Status of which we are talking.

[2]We also use the symbols of the same shape as logical constants of Ontology in meta-language for naming these constants.

[3]This variables are differentiated by upper case numbers, since the lower case number informs us about the number of arguments of these functors.

object which does not actually exist, but either existed or is going to exist, the name is not empty. However, this assumption is not an essential one. We could have assumed the opposite, without any loss of accuracy. *Shared* names, each of which names more than one object (e.g. 'egg'). *Fictitious* names, i.e. expressions which regards their syntax behave like shared or unshared names, but which do not name anything (e.g. 'Gandalf', 'Unicorn').[4]

2.1. *LEO*-Languages

We start with syntactical definitions of languages for which LEO-systems can be built. We shall start from the languages without variables, which are the simplest cases of LEO-languages. For convenience, we shall not distinguish between languages and its algebras of expressions. We introduce the notion of LEO-language without variables ($CLEO$): [5]

2.1.1. *CLEO*-Languages

Definition 2.1. $L \in CLEO \equiv L = \langle S_C^L, N_C^L, \varepsilon, \wedge, \neg\rangle$
where N_C^L is the set of name constants (it does not matter: shared, unshared, or fictitious)[6], ε is a primitive functor of the category $\frac{s}{n_1,n_2}$, $\neg, \wedge$ are classical extensional functors of Sentential Calculus, here treated as primitive ones. S_C^L is the set of well built sentences, being the least set which fulfills the following conditions:[7]

1. $\ulcorner\pi_1\urcorner, \ulcorner\pi_2\urcorner \in N_C^L \rightarrow \ulcorner\varepsilon\langle\pi_1, \pi_2\rangle\urcorner \in S_C^L$
2. $\ulcorner\varphi\urcorner, \ulcorner\psi\urcorner \in S_C^L \rightarrow \ulcorner\neg\varphi\urcorner, \ulcorner\varphi \wedge \psi\urcorner \in S_C^L$

We introduce the notion of a model for $CLEO$-languages.

[4] The conduct of this distinction is due to [2].

[5] 'C' in '$CLEO$' coming from 'Constants'.

[6] As the elements of N_C (and later on, of N_{CV}, to be defined) in a language we will use $\{c, d, c_1, d_1, \ldots, c_n, d_n\}$.

[7] We use quasi-quotation marks, i.e. the expression: $\ulcorner\varphi\urcorner$ is the name of the sentence ϕ.

Definition 2.2. $\mathcal{M}$ is a model for $L \in CLEO \equiv \mathcal{M} = OBJ$ where OBJ is a set of objects.

We introduce the *function of extension of names*, from N_C^L into 2^{OBJ}. The set of such a functions is EXT_L^{OBJ}. In our further deliberations we will often assume that the model and language are settled, and denote the extension of π simply as '$Ext(\pi)$'. We will also use variables representing extension functions of L in $\mathcal{M}$, denoted by $f_L^{\mathcal{M}}$, or f_L^{OBJ}.

Definition 2.3. $\mathcal{I}$ is an interpretation of $L \in CLEO \equiv \mathcal{I} = \langle \mathcal{M}, f_L^{\mathcal{M}} \rangle$.

The *Valuation function – $Val_L^{\mathcal{I}}$ – of a CLEO language, say L, in an interpretation* $\mathcal{I}$ is a function that maps S_C^L onto[8] $\{0, 1\}$ in the way defined below (Let $\mathcal{I}$ be $\langle OBJ, f_L^{\mathcal{M}} \rangle$):

Definition 2.4. $Val_L^{\mathcal{I}}$ is the function from S_C^L onto $\{0, 1\}$, satisfying the conditions:

1. $Val_L^{\mathcal{I}}(\ulcorner \varepsilon\langle \pi_1, \pi_2 \rangle \urcorner) = 1 \equiv \exists! x \in f_L^{\mathcal{M}}(\pi_1) \wedge f_L^{\mathcal{M}}(\pi_1) \subset f_L^{\mathcal{M}}(\pi_2)$
2. $\ulcorner \varphi \urcorner \in S_C^L \rightarrow [Val_L^{\mathcal{I}}(\ulcorner \neg\varphi \urcorner) = 1 \equiv Val_L^{\mathcal{I}}(\ulcorner \varphi \urcorner) = 0]$
3. $\ulcorner \varphi \urcorner, \ulcorner \psi \urcorner \in S_C^L \rightarrow [Val_L^{\mathcal{I}}(\ulcorner \varphi \wedge \psi \urcorner) = 1 \equiv Val_L^{\mathcal{I}}(\ulcorner \varphi \urcorner) = 1 \wedge Val_L^{\mathcal{I}}(\ulcorner \psi \urcorner) = 1]$

Now, obtaining the definition of *truth in a given interpretation* ($T_{\mathcal{I}}$) is trivial:

Definition 2.5. $T_{\mathcal{I}}(\ulcorner \varphi \urcorner) \equiv Val_L^{\mathcal{I}}(\ulcorner \varphi \urcorner) = 1$

2.1.2. $CAVELEO$-Languages

Now we extend our languages, allowing it to contain name constants and variables. We define the set of $CAVELEO$-languages.[9]

Definition 2.6. $L \in CAVELEO \equiv L = \langle FS_{CV}^L, NV_{CV}^L, \varepsilon, \wedge, \neg, \forall \rangle$ where NV_{CV}^L is the union of N_{CV} (which is the set of name constants of L) and V_{CV} (which is the set of name variables of L).[10] The functor

[8] We could as well have said: 'into'. Since we have closed the set S_C^L under the operation of classical negation, it makes no difference, the result is the same.

[9] '$CAVE$' in '$CAVELEO$' coming from 'Constants And VariablEs'.

[10] As elements of V_{CV} (and later on, of V_V, to be defined) in a language we will use $\{a, b, a_1, b_1, \ldots, a_n, b_n\}$.

ε is a primitive functor of category $\frac{s}{n_1,n_2}$, $\neg$, $\wedge$ are classical extensional functors of Sentential Calculus, here treated as primitive ones, $\forall$ is simply the universal quantifier (it did not occur in $CLEO$-languages, since there were no variables to quantify over).

FS^L_{CV} is the union of S^L_{CV} (which is the set of sentences of L), and F^L_{CV} (which is the set of propositional formulas of L). It means that FS^L_{CV} is the least set satisfying the following conditions:

1. $\ulcorner\alpha_1\urcorner, \ulcorner\alpha_2\urcorner \in NV^L_{CV} \rightarrow \ulcorner\varepsilon\langle\alpha_1, \alpha_2\rangle\urcorner \in FS^L_{CV}$
2. $\ulcorner\chi_1\urcorner, \ulcorner\chi_2\urcorner \in FS^L_{CV} \rightarrow \ulcorner\neg\chi_1\urcorner, \ulcorner\chi_1 \wedge \chi_2\urcorner \in FS^L_{CV}$
3. $\ulcorner\mu\urcorner \in V^L_{CV} \wedge \ulcorner\chi\urcorner \in FS^L_{CV} \rightarrow \ulcorner\forall_\mu\chi\urcorner \in FS^L_{CV}$

The definition of model for $L \in CAVELEO$ is the same, as before (definition 2.2 on page 18). Similar situation occurs with respect to the extension function. It maps N^L_{CV} into 2^{OBJ}. The interpretation of a $CAVELEO$-language consists also in giving the model and extension function.

Some difficulties arise, when we want to consider the valuation of variables, and the truth of expressions containing variables. For we can either emphasize that they are NAME variables, or that they are name VARIABLES. The question is: should we valuate a name variable *via* names, or not?

If we choose the first option, consequently, we can allow only these valuations which can be 'obtained' by means of substitution of name variables by names as well. Namely, we must agree that (the extension function $f^{\mathcal{M}}_L$ is given), when we understand a valuation of name variables V^L_{CV} as a sequence $\langle A_1, \ldots, A_n\rangle = A^u$ of n elements of 2^{OBJ}, we have to exclude from possible valuations such tuples A^u for which $\exists_{A_i}\neg\exists_{\pi\in N^L_{CV}}[A_i = f^{\mathcal{M}}_L(\pi)]$.[11] On the other hand, if we choose the second option, we put no restriction on A^u, but accordingly concede that there are such valuations of name variables for which there are no corresponding names. We could avoid this difficulty by the simple assumption that $\forall_{L\in CAVELEO}\forall_{A_i\in 2^{OBJ}}\exists_{\pi\in N^L_{CV}}[A_i = f^{\mathcal{M}}_L(\mu)]$. Unfortunately, languages which do not fulfill this condition seem to be quite legitimate objects of investigation.

[11] We have loosely said, that a set A_i belongs to A^u; we meant that it is an element of the sequence, obviously.

For convenience, we have decided to define the valuation of variables for $CAVELEO$-languages in accordance to the first option, and to leave the most general concept of valuation for $VALEO$-languages (to be defined), which do not contain name constants. The interesting result is that some sentences built by preceding a sentential formula by a universal quantification can be true in a $CAVELEO$-language L just because of the nature of the set of names of L.[12]

To any name we can attribute a set that is its extension (i.e. denotation). In this way, every tautology (or valid expression) in the wider sense of valuation is a tautology (valid formula) in the narrower sense of valuation.

The question is, as we have said, whether the implication in the other direction is true. The answer would be simple, if we assumed the mentioned additional condition. Practically, it seems, however, that we are lacking names. As Ajdukiewicz argues [1, p. 138]:

> Names of each language divide into simple and composed. There is always a finite number of the simple ones, the composite names are always finite combinations of simple names, hence there are $\aleph_0$ names.

If we simply take a universe containing the set of natural numbers (or any other universe of the power $\aleph_0$), according to Cantor's theorem, the number of subsets of the universe will be greater than $\aleph_0$.

Nevertheless, we can claim the following:

Theorem 2.7. *If language L fulfills the following requirement:*

> *For any formula of elementary theory of numbers, if this formula contains exactly one free variable, there is in L a general name, extension of which is identical with the set of numbers satisfying this formula.*[13]

then it is true, that any formula of L valid in the lexical sense, is valid in the semantic sense.

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[12]For instance, if we have $OBJ = \{1,2,3\}$, $N^L_{CV} = \{c,d\}$, $f^{\mathcal{M}}_L(c) = \{1,2\}$, $f^{\mathcal{M}}_L(d) = \{3\}$, it is true in this interpretation that $\forall_{a,b}(\neg(\varepsilon\langle a,b\rangle \wedge \varepsilon\langle b,a\rangle) \rightarrow \neg\varepsilon\langle a,b\rangle)$.

[13]Elementary theory of numbers is what we can say about natural numbers in terms of addition, multiplication, identity, sentence connectors, variables representing natural numbers, and quantifiers binding them, without introducing the notion of set.

The full proof is to be found in [3].

We define the notion of valuation of name variables in $CAVELEO$-languages.

Definition 2.8. The valuation of $V^L_{CV} = \{\mu_1, \ldots, \mu_k\}$ is a sequence: $\langle A_1, \ldots, A_k \rangle = A^u$ of elements of 2^{OBJ} such that $\forall_{A_i} \exists_{\pi \in N^L_{CV}} [A_i = f^{\mathcal{M}}_L(\pi)]$.

The value of μ_i in an interpretation A^u will be denoted as $A^u(\mu_i)$, or simply A_i.

We define the notion of *Satisfaction*:

Definition 2.9. We assume, that the sequences: of names and name variables are fixed.

1. Sentence $\ulcorner \varepsilon\langle \pi_1, \pi_2 \rangle \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\exists!_x x \in f^{\mathcal{M}}_L(\pi_1) \wedge f^{\mathcal{M}}_L(\pi_1) \subset f^{\mathcal{M}}_L(\pi_2)$. It is obvious, that, since $\ulcorner \varepsilon\langle \pi_1, \pi_2 \rangle \urcorner$ does not contain variables (or, in other words, 'A^u' does not occur *in definiente*), if there is at least one A^u satisfying the given sentence in $\mathcal{I}$, this sentence is satisfied by any valuation in $\mathcal{I}$.

2. Let us consider such an expression $\ulcorner \varepsilon\langle \alpha_i, \alpha_j \rangle \urcorner$, in which there is at least one name variable. Obviously, there are three cases:

   (a) $\ulcorner \varepsilon\langle \pi_i, \mu_k \rangle \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\exists!_x x \in f^{\mathcal{M}}_L(\pi_i) \wedge f^{\mathcal{M}}_L(\pi_i) \subset A_k$.

   (b) $\ulcorner \varepsilon\langle \mu_k, \pi_i \rangle \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\exists!_x x \in A_k \wedge A_k \subset f^{\mathcal{M}}_L(\pi_i)$.

   (c) $\ulcorner \varepsilon\langle \mu_k, \mu_i \rangle \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\exists!_x x \in A_k \wedge A_k \subset A_i$.

3. $\ulcorner \neg\chi \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\ulcorner \chi \urcorner$ is not satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u.

4. $\ulcorner \chi_i \wedge_j \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u *if and only if* $\ulcorner \chi_i \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u and $\ulcorner \chi_j \urcorner$ is satisfied in $\mathcal{I} = \langle \mathcal{M}, f^{\mathcal{M}}_L \rangle$ by a valuation A^u.

5. $\lceil\forall_{\mu_k}\chi\rceil$ is satisfied in $\mathcal{I} = \langle\mathcal{M}, f_L^{\mathcal{M}}\rangle$ by a valuation A^u *if and only if* $\lceil\chi\rceil$ is satisfied in $\mathcal{I} = \langle\mathcal{M}, f_L^{\mathcal{M}}\rangle$ by any possible valuation A^d which differs from A^u at most on $k-th$ place.

We define the notion of truth in an interpretation ($T_{\mathcal{I}}$):

Definition 2.10. $T_{\mathcal{I}}(\lceil\chi\rceil) \equiv \forall_u[\chi$ is satisfied by A^u in $I]$

Given sentential expression is true in an interpretation, if it is satisfied by every possible in this interpretation valuation of (its) name variables.

Obviously, we can define validity as being true in any interpretation.

2.1.3. $VALEO$-Languages

We introduce languages without name constants, but containing name variables. We define the set of $VALEO$-languages.[14]

Definition 2.11. $L \in VALEO \equiv L = \langle FS_V^L, V_V^L, \varepsilon, \wedge, \neg, \forall\rangle$ whereas V_V^L is the set of name variables of L, ε is a primitive functor of category $\frac{s}{n_1,n_2}$; $\neg$, $\wedge$ are classical extensional functors of Sentential Calculus, here treated as primitive ones, $\forall$ is simply the universal quantifier.

FS_V^L is the union of S_V^L (which is the set of sentences of L [15]), and F_V^L (which is the set of propositional formulas of L). It means that FS_V^L is the least set satisfying the following conditions:

1. $\lceil\mu_1\rceil, \lceil\mu_2\rceil \in V_V^L \rightarrow \lceil\varepsilon\langle\mu_1, \mu_2\rangle\rceil \in FS_{CV}^L$
2. $\lceil\chi_1\rceil, \lceil\chi_2\rceil \in FS_V^L \rightarrow \lceil\neg\chi_1\rceil, \lceil\chi_1 \wedge \chi_2\rceil \in FS_V^L$
3. $\lceil\mu\rceil \in V_V^L \wedge \lceil\chi\rceil \in FS_V^L \rightarrow \lceil\forall_\mu\chi\rceil \in FS_V^L$

The definition of model is the same, as definition 2.2 on page 18:

Definition 2.12. $\mathcal{M}$ is a model for $L \in VALEO \equiv \mathcal{M} = OBJ$ where OBJ is a set of objects.

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[14] 'VA' in '$VALEO$' coming from 'VAriables'.

[15] Nota bene, though there are no names, it is possible to obtain sentences from formulas by use of quantifier.

Since we use no names, we shall not need the notion of extension of a name. The interpretation of a $VALEO$-language consists only in giving a model. Therefore, we shall not define interpretation (model will suffice). We define the notion of valuation of name variables in $VALEO$-languages.

Definition 2.13. The valuation of $V_V^L = \{\mu_1, \ldots, \mu_k\}$is a sequence:

$$\langle A_1, \ldots, A_k \rangle = A^u \text{ of elements of } 2^{OBJ}$$

The value of μ_i in an interpretation A^u will be denoted as $A^u(\mu_i)$, or simply A_i.

We define the notion of *Satisfaction*:

Definition 2.14. We assume, that the sequence of variables is fixed.

1. $\lceil \varepsilon\langle \mu_k, \mu_i \rangle \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u *if and only if* $\exists!_x x \in A_k \wedge A_k \subset A_i$.

2. $\lceil \neg \chi \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u *if and only if* $\lceil \chi \rceil$ is not satisfied in $\mathcal{M}$ by a valuation A^u.

3. $\lceil \chi_i \wedge \chi_j \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u *if and only if* $\lceil \chi_i \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u and $\lceil \chi_j \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u.

4. $\lceil \forall_{\mu_k} \chi \rceil$ is satisfied in $\mathcal{M}$ by a valuation A^u *if and only if* $\lceil \chi \rceil$ is satisfied in $\mathcal{M}$ by any valuation A^d which differs from A^u at most on $k - th$ place.

We define the notion of truth in model ($T_{\mathcal{M}}$):

Definition 2.15. $T_{\mathcal{M}}(\lceil \chi \rceil) \equiv \forall_u[\chi$ is satisfied in $\mathcal{M}$ by $A^u]$

Given sentence expression is true in an interpretation, if it is satisfied by every possible in this model valuation of name variables. Validity is defined as truth in any model.

2.2. The so-called Nominalism of LEO-systems

Leśniewski, in fact, was a nominalist. Hence, sometimes, his systems (systems formulated in *LEO*-languages, *a fortiori*) are believed to be nominalistic. However, the deliberations hitherto led show us only that what suffices as a model of given *LEO*-language, is a set of objects.

This statement obliges use neither to accept, nor to refute nominalism. We have decided neither what kind of objects can belong to the set OBJ, nor what object must belong to this set.

Moreover, it is far from being clear, that belonging to a model is to be interpreted as real existence. Thus, even, if we had decided, that a model $\mathcal{M}$ of an *LEO*-language L can contain only objects of a particular sort, still it would not force us to accept any claim either about real existence of anything, or about the lack of it.

3. Notion of Semantic Status (SeS)

3.1. Intuitions

There are some specific functors which we will be considering in this paper. Those are *Ontological Functors* (OF). They are those specific functors which distinguish *Elementary Ontology* from *Prototetics.* As it is obvious, these functors are of syntactical category $\frac{s}{n_1,\ldots,n_c}$.

However, this information does not suffice for distinguishing OF-s from other functors of the same syntactical categories. For example, we would consider the expression ‘is’ in: ‘Socrates is mortal.’ an OF. This functor is of the syntactical category $\frac{s}{n,n}$; but we can as well find a functor of the same syntactical category, which surely is not an OF, e.g. the expression ‘loves’ in ‘John loves Mary.’.

Hence we need some other condition (or a set of conditions), which would not only be necessary, but also sufficient for a functor to be an OF. In what follows, we shall fulfill this requirement.

The state of affairs affirmed by propositions built from an OF and its name arguments is called the (possible) semantic status (SeS) of this arguments.

3.2. 1-place SeS-es

Definition 3.1. Semantic Status of a given name π is I.i. $(1 \geq i \geq 3)$, whereas i is equal (respectively) to:

$$\begin{cases} 1 & iff \quad \exists! x[x \in Ext(\pi)] \\ 2 & iff \quad \exists x[x \in Ext(\pi)] \wedge \neg\exists! x[x \in Ext(\pi)] \\ 3 & iff \quad \neg\exists x[x \in Ext(\pi)] \end{cases}$$

Instead of saying: 'an SeS which is k-place' we will use lower case numbers: 'SeS_k' is a name (shared) with $Ext(SeS_k)$ identical with the set of all k-place SeS-es.[16]

Example 3.2. The SeS_1 of the name *Socrates* is I.1.. △[17]

Instead of saying 'the SeS_k of k names (in given order) $\langle \pi_1, \ldots, \pi_k \rangle$' we will simply write: '$SeS_k\langle \pi_1, \ldots, \pi_k \rangle$'. We also, where there will be no danger of ambiguity, will use the notation of SeS_k as a function of k arguments $\langle \pi_1, \ldots, \pi_k \rangle$ with the Semantic Status of $\langle \pi_1, \ldots, \pi_k \rangle$ as value.

3.3. 2-place SeS

Now we proceed to defining all possible 2-place SeS-es.

From now on, if we are talking about a semantic status of k names, it is to be assumed, that we consider the order of these names important. We sometimes write instead of 'a SeS of k names (in order)' simply 'SeS of k names', just for convenience.

Definition 3.3. Semantic Status of two names (in order) $\langle \pi_1, \pi_2 \rangle$ is $II.i$ $(1 \geq i \geq 16)$, where i is equal (respectively) to:

[16] For clarity of presentation, we will sometimes use the abbreviations introduced for the terms defined, just as if they were nouns. We will also use their singular and plural forms. The singular forms are identical with abbreviations themselves; The plural ones are constructed by adding the endings: -s, or -es. Strictly speaking, our definitions of SeS-es are definitions of functions. However, when we use our symbols otherwise, it is made for the sake of presentation, and, we believe, there is no danger od ambiguity.

[17] We will use the symbol '△' as indicating the end of an example.

$$
\begin{cases}
1 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.1. \wedge Ext(\pi_1) = Ext(\pi_2) \\
2 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.1. \wedge Ext(\pi_1) \neq Ext(\pi_2) \\
3 \text{ iff } SeS_1\langle\pi_1\rangle = I.1. \wedge SeS_1\langle\pi_2\rangle = I.2. \wedge Ext(\pi_1) \subset Ext(\pi_2) \\
4 \text{ iff } SeS_2\langle\pi_2,\pi_1\rangle = II.3. \\
5 \text{ iff } SeS_1\langle\pi_1\rangle = I.1. \wedge SeS_1\langle\pi_2\rangle = I.2. \wedge Ext(\pi_1) \not\subset Ext(\pi_2) \\
6 \text{ iff } SeS_2\langle\pi_2,\pi_1\rangle = II.5. \\
7 \text{ iff } SeS_1\langle\pi_1\rangle = I.1. \wedge SeS_1\langle\pi_2\rangle = I.3. \\
8 \text{ iff } SeS_2\langle\pi_2,\pi_1\rangle = II.7. \\
9 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.2. \wedge Ext(\pi_1) = Ext(\pi_2) \\
10 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.2. \wedge Ext(\pi_1) \subset Ext(\pi_2) \wedge \\
\qquad \wedge Ext(\pi_1) \neq Ext(\pi_2) \\
11 \text{ iff } SeS_2\langle\pi_2,\pi_1\rangle = II.10. \\
12 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.2. \wedge \exists x[x \in Ext(\pi_1) \wedge \\
\qquad \wedge x \notin Ext(\pi_2)] \wedge \exists x[x \in Ext(\pi_2) \wedge x \notin Ext(\pi_1)] \wedge \\
\qquad \wedge \exists x[x \in Ext(\pi_1) \wedge x \in Ext(\pi_2)] \\
13 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.2. \wedge \neg\exists x[x \in Ext(\pi_1) \wedge \\
\qquad \wedge x \in Ext(\pi_2)] \\
14 \text{ iff } SeS_1\langle\pi_1\rangle = I.2. \wedge \\
\qquad \wedge SeS_1\langle\pi_2\rangle = I.3. \\
15 \text{ iff } SeS_2\langle\pi_2,\pi_1\rangle = II.14. \\
16 \text{ iff } SeS_1\langle\pi_1\rangle = SeS_1\langle\pi_2\rangle = I.3.
\end{cases}
$$

3.4. Ontological Table

The $SeS_1 - es$ and SeS_2-es hitherto defined can, perhaps, be better grasped, if we apply the graphical method of representing them. The method itself was used in [2, p. 128]. Lejewski however, has not defined the notion of *Semantic Status* and has finished his semantic considerations on presenting the Ontological Table, which, for us, is just a point of departure for further investigations. Nevertheless, we present the Table, just as it occurs in [2], for convenience of the reader. By a shaded circle we represent the only object named by an unshared name. By an unshaded circle we represent the many objects each of which is named by a shared name. No circle will be used in case of fictitious name.

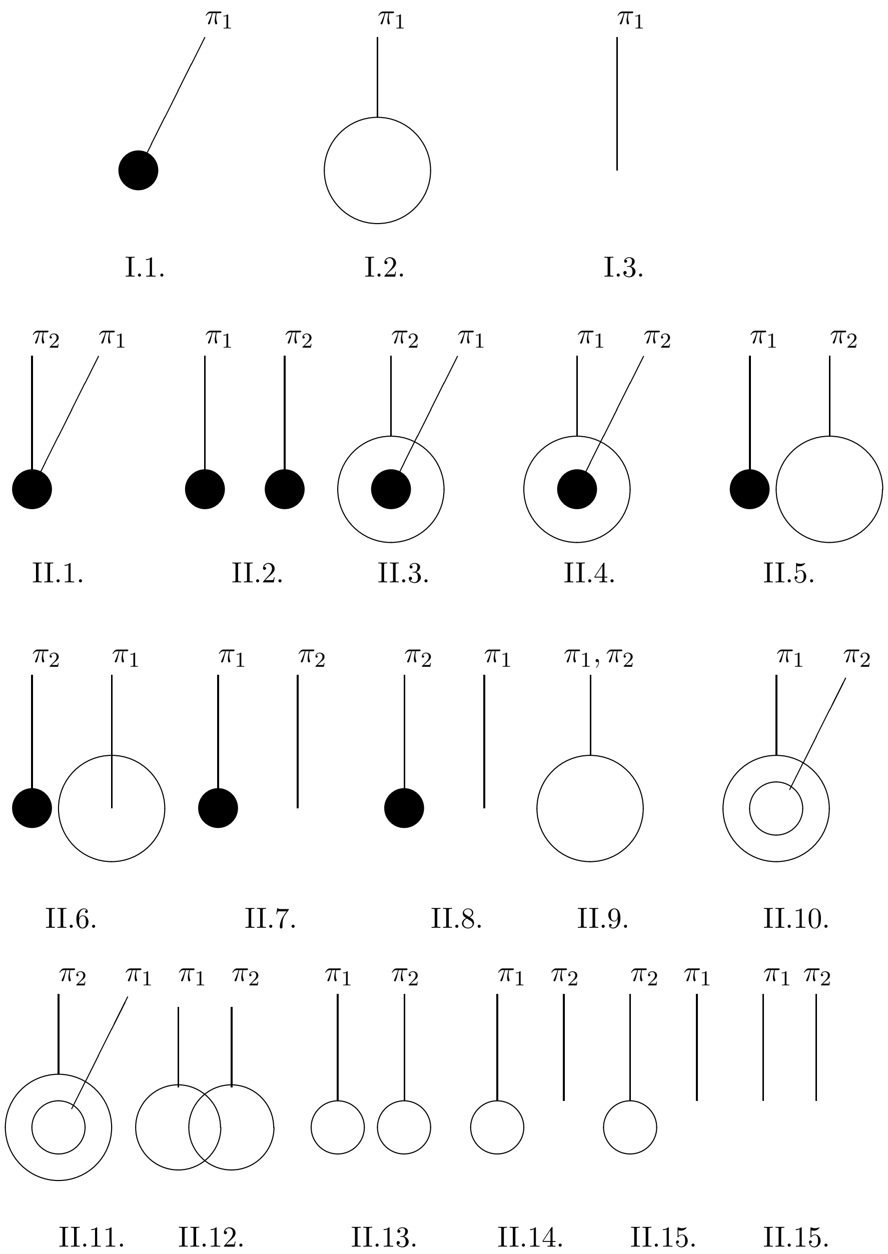

Figure 1: The Ontological Table

3.5. Identity, Union and Intersection of SeS-es

We define the relation of identity in the set of SeS-es. Two SeS-es, say σ_k^1, σ_m^2, can remain in this relation only if $k = m$:

Definition 3.4.
$$\begin{aligned}&\sigma_k^1 = \sigma_m^2 \equiv m = k \wedge \\ &\wedge \forall_{\pi_1^1,\ldots,\pi_k^1}[SeS_k\langle\pi_1^1,\ldots,\pi_k^1\rangle = \\ &= \sigma_k^1\langle\pi_1^1,\ldots,\pi_k^1\rangle \equiv \\ &\equiv SeS_k\langle\pi_1^1,\ldots,\pi_k^1\rangle = \sigma_m^2\langle\pi_1^1,\ldots,\pi_k^1\rangle]\end{aligned}$$

Now we define the notion of *union of SeS-es*. Let us consider two sequences of names: $\pi_1^1,\ldots,\pi_k^1$ and $\pi_1^2,\ldots,\pi_m^2$. Let $\pi_1^{1,2},\ldots,\pi_c^{1,2}$ be all different name variables among $\pi_1^1,\ldots,\pi_k^1$ and $\pi_1^2,\ldots,\pi_m^2$.

Definition 3.5.
$$\begin{aligned}&SeS_c\langle\pi_1^{1,2},\ldots,\pi_c^{1,2}\rangle = [\sigma_k^1\langle\pi_1^1,\ldots,\pi_k^1\rangle]\cup \\ &\cup[\sigma_m^2\langle\pi_1^2,\ldots,\pi_m^2\rangle] \equiv SeS_k\langle\pi_1^1,\ldots,\pi_k^1\rangle = \\ &= \sigma_k^1\langle\pi_1^1,\ldots,\pi_k^1\rangle \vee [SeS_m\langle\pi_1^2,\ldots,\pi_m^2\rangle = \\ &= \sigma_m^2\langle\pi_1^2,\ldots,\pi_m^2\rangle]\end{aligned}$$

We define the notion of *intersection of SeS-es*:

Definition 3.6.
$$\begin{aligned}&SeS_{k+m}\langle\pi_1^{1,2},\ldots,\pi_c^{1,2}\rangle = [\sigma_k^1\langle\pi_1^1,\ldots,\pi_k^1\rangle]\cap \\ &\cap[\sigma_m^2\langle\pi_1^2,\ldots,\pi_m^2\rangle] \equiv SeS_k\langle\pi_1^1,\ldots,\pi_k^1\rangle = \\ &= \sigma_k^1\langle\pi_1^1,\ldots,\pi_k^1\rangle \wedge [SeS_m\langle\pi_1^2,\ldots,\pi_m^2\rangle = \\ &= \sigma_m^2\langle\pi_1^2,\ldots,\pi_m^2\rangle]\end{aligned}$$

3.6. ($n \geq 3$)-place SeS

We are going to introduce the notion of SeS of $n \geq 3$ names. We precede the formal definition by some intuitive deliberations. The SeS_1-es and SeS_2-es are states of affairs which concern names and pairs of names. Knowing the SeS_1 of a name, we know, whether there are no objects named by this name, or there is exactly one such an object, or there are more than one such objects. Knowing the SeS_2 of any given pair of names, we not only know, what the SeS_1 of each of these names is, but also, in what relation to each other remain the sets of their designates.

It is however quite natural, that the number of names which may be taken under consideration with respect to their SeS should not come to an end with 2. We can ask, in what 3-place semantic (to say it loosely) relation given three names remain.

Intuitively, we would allow all n-place (where $n \in \mathcal{N}$) relations between names to be n-place SeS-es, as far, as they would be representable by means of diagrams similar to these used in the Ontological Table in the Figure 1.

Quite helpful in defining $SeS_{n\geq 3}$, say σ^1, seems to be the fact, that if we have $\sigma^1_{n\geq 3}\langle \pi_1, \ldots, \pi_{n\geq 3}\rangle$, we know all SeS_2 of all pairs from $\{\pi_1, \ldots, \pi_{n\geq 3}\} \times \{\pi_1, \ldots, \pi_{n\geq 3}\}$, i.e. from $\{\pi_1, \ldots, \pi_{n\geq 3}\}^2$. Also, it seems to work in the other direction: if we know all SeS_2 of all pairs from $\{\pi_1, \ldots, \pi_{n\geq 3}\}^2$, we know what the $\sigma^1_{n\geq 3}\langle \pi_1, \ldots, \pi_{n\geq 3}\rangle$ is. Hence, (according to our intuitive deliberations and the definition 3.4 of identity) we define:

Definition 3.7. $\sigma^1 = SeS_{k\geq 3}\langle \pi_1, \ldots, \pi_{k\geq 3}\rangle \equiv$
$$\equiv \sigma^1 = \bigcap \left\{ \sigma^2_2\langle \pi_i, \pi_j\rangle : \langle \pi_i, \pi_j\rangle \in \{\pi_1, \ldots, \pi_{k\geq 3}\}^2 \right\}$$

In other words, a SeS is a SeS of more than two names *if and only if* it is identical with intersection of SeS_2-es of all pairs of names which were taken under consideration.

Definition 3.8. σ is a SeS *if and only if* it is a SeS_k, for some $k \in \mathcal{N}$

3.7. Theorems on SeS-es

Corollary 3.9. $\forall \pi[SeS_1(\pi) = I.1. \vee SeS_1(\pi) = I.2. \vee SeS_1(\pi) = I.3.]$

Corollary 3.9 claims that for every name π, there is among I.1., I.2., I.3. at least one Semantic State of π.

Proof. Let B be a set-variable. It is true that:

$$\forall B[\exists! x[x \in B] \vee \exists x[x \in B] \wedge \neg\exists! x[x \in B] \vee \neg\exists x[x \in B]]$$

Since the Ext function takes sets as values, it is also the case that for all π:

$$\exists! x[x \in Ext(\pi)] \vee \exists x[x \in Ext(\pi)] \wedge \neg\exists! x[x \in Ext(\pi)] \vee \neg\exists x[x \in Ext(\pi)]$$

If $\exists! x[x \in Ext(\pi)]$, then, according to the definition given, $SeS_1(\pi) = I.1.$. If $\exists x[x \in Ext(\pi)] \wedge \neg\exists! x[x \in Ext(\pi)]$, and being so, according to the definition given, $SeS_1(\pi) = I.2.$. If $\neg\exists\ x[x \in Ext(\pi)]$, according to the definition given, then $SeS_1(\pi) = I.3.$. The formula: $(p \vee q \vee r) \rightarrow [(p \rightarrow s) \wedge (q \rightarrow t) \wedge (r \rightarrow u) \rightarrow (s \vee t \vee u)]$ is a tautology of Sentential Calculus. By a proper substitution, *modo ponendo ponente* we obtain the demanded corollary. □

In the similar manner, using set theory and the introduced semantics, we can easily prove the following few claims:

Corollary 3.10. $\forall \pi \neg \exists_{i \neq j} [SeS_1 \langle \pi \rangle = I.i. \wedge SeS_1 \langle \pi \rangle = I.j.]$

Lemma 3.11. $\forall \pi \exists !_i [SeS_1 \langle \pi \rangle = I.i.]$

Corollary 3.12. $\forall \pi_1, \pi_2 \exists_{1 \leq i \leq 16} [SeS_2 \langle \pi_1, \pi_2 \rangle = II.i.]$

Corollary 3.13. $\forall \pi_1, \pi_2 \forall_{1 \leq i,j \leq 16} [SeS_2 \langle \pi_1, \pi_2 \rangle = II.i. \wedge$
$\wedge SeS_2 \langle \pi_1, \pi_2 \rangle = II.j. \rightarrow i = j]$

Lemma 3.14. $\forall \pi_1, \pi_2 \exists !_{1 \leq i \leq 16} [SeS_2 \langle \pi_1, \pi_2 \rangle = II.i.]$

Corollary 3.10 tells us that for every name π there is among I.1., I.2., I.3. at most one Semantic Status of π. Lemma 3.11 informs that each name has exactly one one-place Semantic Status. Corollary 3.12 says that for each two names (in order, obviously) their SeS_2 is identical with at least one of the $SeS_2 - es$ already defined. Corollary 3.13 claims that for every pair of names there is at most one SeS_2 such that it is the SeS of these names. Lemma 3.14 says, that for every two names there is exactly one SeS_2 among I.1. - 1.16. which is the SeS_2 of these names.

4. Ontological Functors (OF)

4.1. 1-Place Ontological Functors ($OF_1 - s$)

Definition 4.1. Functor δ_1 is a SOF_1 *if and only if*

$$\forall \pi \exists !_{1 \leq i \leq 3} [\delta_1 \langle \pi \rangle \equiv SeS_1 \langle \pi \rangle = I.i.]$$

By writing '$i \neq j \neq k$' etc. we mean that i, j, k are distinct from each other.

Definition 4.2. Functor δ_1 is a POF_1 *if and only if*

$$\exists_{1\leq i\neq j\neq\ldots\leq 3}[\delta_1\langle\pi\rangle \equiv \underbrace{SeS_1\langle\pi\rangle = I.i. \vee \ldots \vee SeS_1\langle\pi\rangle = I.j.}_{\text{the number of disjuncts is 2 or 3}}]$$

There is also one specific 2-place functor, neither SOF_2, nor POF_2, namely $Falsum$ - F_2. With its name arguments it gives a false sentence.[18]

Definition 4.3. A functor δ is an OF_1 *if and only if*

it is either a SOF_1, or a POF_1, or F_1

4.2. 2-Place Ontological Functors ($OF_2 - s$)

Definition 4.4. Functor δ_2 is a SOF_2 *if and only if*

$$\forall_{\pi_1,\pi_2}\exists!_{1\leq i\leq 16}[\delta_1\langle\pi_1,\pi_2\rangle \equiv SeS_2\langle\pi_1,\pi_2\rangle = II.i.]$$

Definition 4.5. Functor δ_2 is a POF_2 *if and only if*

$$\exists_{1\leq i\neq j\neq\ldots\leq 16}[\delta_1\langle\pi_1,\pi_2\rangle \equiv SeS_2\langle\pi_1,\pi_2\rangle = II.i.\vee$$
$$\vee\ldots\vee SeS_2\langle\pi_1,\pi_2\rangle = II.j.]$$

There is also one specific 2-place functor, neither SOF_2, nor POF_2, namely $Falsum$ - F_2. With its name arguments it gives a false sentence.

Definition 4.6. Functor δ_2 is an OF_2 *if and only if* δ_2 is either a SOF_2, or a POF_2, or F_2.

4.3. $k \geq 3$-place Ontological Functors

Definition 4.7. Functor $\delta_{k\geq 3}$ is a $SOF_{k\geq 3}$ *if and only if*

$$\forall_{\pi_1,\ldots,\pi_k}\exists!_{\sigma_{k\geq 3}}[SeS_{k\geq 3}\langle\pi_1,\ldots,\pi_k\rangle = \sigma_{k\geq 3} \equiv \delta_{k\geq 3}\langle\pi_1,\ldots,\pi_k\rangle]$$

Definition 4.8. $\delta_{k\geq 3}$ is a $POF_{k\geq 3}$ *if and only if*

$$\exists_{\sigma_k^1\neq\ldots\neq\sigma_k^u}\forall_{\pi_1,\ldots,\pi_k}[\delta_{k\geq 3}\langle\pi_1,\ldots,\pi_k\rangle \equiv$$
$$\equiv \sigma_k^1 = SeS_k\langle\pi_1,\ldots,\pi_k\rangle \vee\ldots\vee \sigma_k^u = SeS_k\langle\pi_1,\ldots,\pi_k\rangle]$$

Definition 4.9. Functor $\delta_{k\geq 3}$ is $F_{k\geq 3}$ (i.e. $falsum$ functor) *if and only if* with its k name arguments, it always gives a false sentence.

[18] Clearly, it does not fit into definitions of SOF-s or POF-s.

Definition 4.10. Functor $\delta_{k\geq 3}$ is an $OF_{k\geq 3}$ *if and only if* it is either a $SOF_{k\geq 3}$, or a $POF_{k\geq 3}$, or $F_{k\geq 3}$.

Obviously, a given functor δ_k is an OF *if and only if* it is either an OF_1, or an OF_2, or an $OF_{k\geq 3}$.

Two k-place OF-s, say δ_k^1, δ_k^2 are identical *if and only if* the truth conditions of sentences obtained by them and their arguments are the same:

Definition 4.11. $\delta_k^1 = \delta_k^2 \equiv \forall_{\pi_1,\ldots,\pi_k}[\delta_k^1\langle\pi_1,\ldots,\pi_k\rangle \equiv \delta_k^2\langle\pi_1,\ldots,\pi_k\rangle]$

5. Defining Ontological Functors

Lemma 5.1. $\overline{\overline{SeS_k}} = \overline{\overline{SOF_k}}$

Proof. There is a function from the set SOF_k onto the set SeS_k. What remains to be shown, is that this function is $1-1$. While defining, we are distinguishing subsets of the set of all functors of $\frac{s}{n_1,\ldots,n_k}$. Now, for any given SeS_k, say σ_k, among all possible k-place functors of category $\frac{s}{n_1,\ldots,n_k}$, there is at least one functor, say δ^1, satisfying the conditions given in the definition of SOF_k, simply because the conditions given are not contradictory. Now, we show that it is unique. Do notice, that if there was any functor with the same truth-conditions, according to the definition 4.11 on p. 32, it would be identical with our δ^1. Next, if it had different truth conditions, it either would not be a SoF_K, or it would be a SoF_K corresponding to an another SeS_1. Hence, for every SeS_k there is exactly one SOF_k corresponding to it. □

In our metalanguage we will use variables already introduced. However, definitions which we will give according to the procedures (these procedures will be described below), will be introduced in an exemplary system of $VALEO$ language, in which name variables are $a, b, a_1, b_1, \ldots, a_n, b_n$. The definitions of OF-s from now on called: 'OF-Definitions' will be equivalences (following the style of Leśniewski[19]). For the purpose of our paper, the form of definition makes no difference.

[19] With the difference, that our definitions will not be preceded by universal quantifier(s). Of course, such an addition, for our purpose would not be essential.

5.1. OF_1-s

As it is discernible, there are exactly three $SOF_1 - s$. In order to systematize them, and to give some hints which allow to understand intuitively the method of defining OF-s further developed, let us construct following table:

$SOF_1 - s$ TABLE			
I.1.	I.2.	I.3.	Functor
1	0	0	*ob*
0	1	0	s (from 'shared')
0	0	1	fi (from 'fictitious')

To each of first three columns of this table there is a corresponding SeS_1. Each line of this table corresponds to an SOF_1. We put '1' in a column of a SeS_1, say σ, in the line of a SOF_1, say δ, to denote that the occurrence of σ of given name, say π, is a sufficient condition of the truth of $\delta\langle\pi\rangle$. We put '0' in the column of δ in the column of σ to denote that the non-occurrence of σ of π is a necessary condition of the truth of $\delta\langle\pi\rangle$.

Hence, we may introduce a convenient general method of referring to k-place functors:

Instruction 1. After construction of an $m+1$-column table, where m is the number of possible different SeS_k , and each column $i-th$ from the left of this table corresponds to the $i-th$ of $SeS_k - es$ (their order to be fixed), and the column $m+1$ is left for placing functor-symbols, every OF_k, say δ, may be represented by a sequence consisting of m elements , each of which is 0 or 1. The $i-th$ element of the sequence is 1 *if and only if* in the $i-th$ column, in the line of δ there is 1. Otherwise, it is 0. Obviously, there are exactly 2^m such sequences.

Thus, the table just given may be extended to include all possible three place sequences of elements of $\{0,1\}$:[20]

[20] Where the names of functors have been already introduced in the history of Ontology, we simply use them. Where there are no such names, we introduce them.

$OF_1 - s$ TABLE			
I.1.	I.2.	I.3.	Functor
1	0	0	*ob*
0	1	0	*s* (from 'shared')
0	0	1	*fi* (from 'fictitious')
0	0	0	F_1 (from 'falsum')
1	1	0	*ex*
1	0	1	*sol*
1	1	1	V_1 (from 'verum')
0	1	1	*nob* (from 'non-object')

Now we define SOF_1 with the use of $\{\wedge, \neg, \forall, \varepsilon\}$ only.

OF-Definition 1. $ob\langle a\rangle \equiv \exists b[\varepsilon\langle a, b\rangle]$ [21]

OF-Definition 2. $s\langle a\rangle \equiv \exists b[\varepsilon\langle b, a\rangle] \wedge \neg\exists b[\varepsilon\langle a, b\rangle]$

OF-Definition 3. $f\langle a\rangle \equiv \forall b, c[\varepsilon\langle b, a\rangle \wedge \varepsilon\langle c, a\rangle \rightarrow$

$$\rightarrow \varepsilon\langle b, c\rangle \wedge \varepsilon\langle c, b\rangle] \wedge \neg\exists b\ [\varepsilon\langle b, a\rangle]$$

Thus we obviously have:

Lemma 5.2. *All SOF_1-s are definable by means of $\{\wedge, \neg, \forall, \varepsilon\}$ as the only logical constants.*

Here is the method of constructing a definition of any OF_1 which is not a SOF_1 by means of SOF_1-s:

Instruction 2. 1. Represent the functor to be defined $\delta_1\langle a\rangle$ by a 3-place sequence according to the INSTRUCTION 1 on page 33.

2. For every place of the sequence here is exactly one formula corresponding to it; namely: '$ob\langle a\rangle$' to the first, '$s\langle a\rangle$' to the second, and '$fi(a)$' to the third.

3. Construct the conjunction of negations of all these three formulas *if and only if* no element of the sequence is 1.

[21] It is important, that (especially particular) quantifiers in our meta-language are interpreted differently from quantifiers in LEO. The first are understood existentially, the second are not.

4. *If and only if* more than one element of the sequence is 1, construct the disjunction of formulas corresponding to these elements.(The case when exactly one element of the sequence is excluded, since this procedure is a procedure of defining OF_1-s which are not $SOF_1 - s$.)

5. As a result of those steps, a formula is obtained. Let it be τ. The formula τ is the right side of the definition. The left side is $\delta_1\langle a\rangle$. Construct the formula $\delta_1\langle a\rangle \equiv \tau$. this is the definition of δ_1.

To make clear the proper understanding of this procedure, we will lead it for one functor:

Example 5.3. Let us consider the functor '*ex*'. It is represented by $\langle 1, 1, 0\rangle$ More than one element of this sequence is 1. Hence, we go to the step 4 and obtain $ob\langle a\rangle \vee s\langle a\rangle$. We obtain the definition:

OF-Definition 4. $ex\langle a\rangle \equiv ob\langle a\rangle \vee s\langle a\rangle$ △

According to the procedure just characterized, we can proceed with the remaining definitions of OF_1-s. Therefore:

Lemma 5.4. *All OF_1-s are definable with the use of 'ε' as the only OF.*

Proof. According to lemma 5.2 on p. 34 'ε' suffices as the only OF for defining all SOF_1-s. As we have seen, all OF_1 not being SOF_1-s are definable by means of SOF_1-s.[22] All SOF_1 have been defined by means of 'ε'. Therefore, we can any given expression (also any definition) containing any OF_1 other than replace by an equivalent formula with 'ε' as the only OF. We can do it also in definitions of OF_1-s, in which we have used SOF_1 *in definientibus*, thus obtaining *definitiones in quarum definientibus* 'ε' is the only OF. □

5.2. OF_2-s

We consider SOF_2-s. As it may be seen from the Ontological Table, there are 16 exactly different SeS_2. Therefore, we can obtain the number of possible OF_2-s. It is equal to the number of SOF_2-s.

[22]Since the usage of non-ontological functors in definitions is obvious, and we are mainly concerned with ontological functors, we omit the phrase: 'as the only OF' where it is obvious from the context.

For convenience, functors for which symbols have not been hitherto introduced in those part of the history of Leśniewski's Ontology which is known to the author of this text, will be symbolized by those sequences of elements of $\{0, 1\}$, which correspond to those functors similarly to the convention introduced for referring to OF_1-s. However, this convention requires an extension. First, we construct a similar table. The understanding of the last column of the table, will be explained in a moment.

$SOF_2 - s$ TABLE																
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14	15	16.	OF
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	=
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	{2}
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	{3}
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	{4}
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	{5}
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	{6}
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	{7}
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	{8}
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	{9}
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	{10}
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	{11}
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	{12}
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	{13}
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	{14}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	{15}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	{16}

Instruction 3. It will be convenient to represent OF_2-s, excluding F_2 and V_2 (which will be simply denoted by symbols 'F_2' and 'V_2'), in some other way than by 16-place sequences (as the instruction 1 on page 33 would suggest). First we construct the table being an extension of the SOF_2-s TABLE to OF_2-s TABLE. The extension consists in adding all possible sequences of 0, 1 as lines of the table. An OF_2, say δ_2, is represented by a set $\{k_1, \ldots, k_m\}$, $16 \geq k_1, k_m \geq 1$, where m is the number of occurrences of symbol '1' in the line of the OF_2- TABLE, which corresponds to this δ_2, and are numbers of columns in which '1' occurs, counting from left, and $\forall_{i,j}[k_i \neq k_j]$.

Clearly, all SOF_2-s are definable by means of ε as the only OF.[23]

[23] I cannot prove it in general, so I simply show it, giving the required definitions below. To grasp the sense of those definitions it suffices to consider definitions hitherto given and The Ontological Table.

OF-Definition 5. $= \langle a, b\rangle \equiv \varepsilon\langle a, b\rangle \wedge \varepsilon\langle b, a\rangle$

OF-Definition 6. $\{2\}\ \langle a, b\rangle \equiv ob\langle a\rangle \wedge ob\langle b\rangle \wedge \neg\varepsilon\langle a, b\rangle$

Example 5.5. The sentence formed from $\{2\}$ (corresponding to the second line of SOF_2-s TABLE) and its name arguments π_1, π_2 has exactly one sufficient truth-condition among SeS-es. Namely, the occurrence of the semantic status II.2. of $\langle \pi_1, \pi_2 \rangle$. Therefore we write '$\{2\}$'. The number of elements of this sequence 'tells us' that there is only one SeS_2, occurrence of which is the sufficient condition of the true of the sentence under consideration. The number '2' inside the brackets tells us which of all SeS_2-es it is. △

Lemma 5.6. *All SOF_2-s are definable by means of 'ε' as the only OF.*

All POF_2-s can be exhaustively listed out by listing all possible 16-place

---

OF-Definition 7. $\{3\}\langle a, b\rangle \equiv ob\langle a\rangle \wedge s\langle b\rangle \wedge \varepsilon\langle a, b\rangle$

OF-Definition 8. $\{4\}\langle a, b\rangle \equiv s\langle a\rangle \wedge ob\langle b\rangle \wedge \varepsilon\langle b, a\rangle$

OF-Definition 9. $\{5\}\langle a, b\rangle \equiv ob\langle a\rangle \wedge s\langle b\rangle \wedge \neg\varepsilon\langle a, b\rangle$

OF-Definition 10. $\{6\}\langle a, b\rangle \equiv s\langle a\rangle \wedge ob\langle b\rangle \wedge \neg\varepsilon\langle b, a\rangle$

OF-Definition 11. $\{7\}\langle a, b\rangle \equiv ob\langle a\rangle \wedge fi\langle b\rangle$

OF-Definition 12. $\{8\}\langle a, b\rangle \equiv fi\langle a\rangle \wedge ob\langle b\rangle$

OF-Definition 13. $\{9\}\langle a, b\rangle \equiv s\langle a\rangle \wedge s\langle b\rangle \wedge \forall_c[\varepsilon\langle c, a\rangle \equiv \varepsilon\langle c, b\rangle]$

OF-Definition 14. $\{10\}\langle a, b\rangle \equiv s\langle a\rangle \wedge s\langle b\rangle \wedge \forall_c[\varepsilon\langle c, a\rangle \rightarrow \varepsilon\langle c, b\rangle] \wedge \neg\forall_c[\varepsilon\langle c, b\rangle \equiv \varepsilon\langle c, a\rangle]$

OF-Definition 15. $\{11\}\langle a, b\rangle \equiv s\langle a\rangle \wedge s\langle b\rangle \wedge \neg\forall_c[\varepsilon\langle c, a\rangle \rightarrow \varepsilon\langle c, b\rangle] \wedge$
$\wedge \forall_c[\varepsilon\langle c, b\rangle \rightarrow \varepsilon\langle c, a\rangle]$

OF-Definition 16. $\{12\}\langle a, b\rangle \equiv s\langle a\rangle \wedge s\langle b\rangle \wedge \exists_{c_1}[\varepsilon\langle c_1, a\rangle \wedge \neg\varepsilon\langle c_1, b\rangle] \wedge$
$\wedge \exists_{c_2}[\varepsilon\langle c_2, a\rangle \wedge \varepsilon\langle c, b\rangle] \wedge \exists_{c_3}[\varepsilon\langle c_3, b\rangle \wedge \neg\varepsilon\langle c, a\rangle]$

OF-Definition 17. $\{13\}\langle a, b\rangle \equiv s\langle a\rangle \wedge s\langle b\rangle \wedge \neg\exists_c[\varepsilon\langle c, a\rangle \wedge \varepsilon\langle c, b\rangle]$

OF-Definition 18. $\{14\}\langle a, b\rangle \equiv s\langle a\rangle \wedge fi\langle b\rangle$

OF-Definition 19. $\{15\}\langle a, b\rangle \equiv fi\langle a\rangle \wedge s\langle b\rangle$

OF-Definition 20. $\{16\}\langle a, b\rangle \equiv fi\langle a\rangle \wedge fi\langle b\rangle$

We define the $falsum$ 2-place functor F_2:

OF-Definition 21. $F_2\langle a, b\rangle \equiv ob\langle a\rangle \wedge \neg ob\langle b\rangle$

It is so called, because with its two name arguments it always yields a false proposition.

sequences of elements of $\{0,1\}$ such that more than one element of sequence is 1. According to instruction 3 on p. 36, it can be represented by a set of a specific kind (already described). We can now use our notation for introducing a general scheme od defining POF_2-s:

Instruction 4. Follow the steps:

1. Represent the functor to be defined, say δ_2, by a set according to instruction 3.
2. For every number $16 \geq i \geq 1$ which may occur in the set such obtained, there is a formula corresponding to it. It is the formula built from SOF_2: '$\{i\}$' and its arguments $\langle ab \rangle$.
3. Any POF_2 is represented by a set $\{k_1, \ldots, k_m\}$, where $16 \geq m \geq 2$. Construct the disjunction of formulas corresponding to numbers occurring in this set.
4. As a result of those steps, a formula is obtained. Let it be τ. This τ is the right side of the definition. The left side is $\delta_2\langle a,b\rangle$. Construct the equivalence : $\delta_2\langle a,b\rangle \equiv \tau$. This will be the definition looked after.

Example 5.7. We proceed to obtain a definition of the functor of strong inclusion which were listed among 14 OF-s by Lejewski in [2, p. 129-130]. This functor usually it is noted by ' $<'$. However, to avoid ambiguity of notation, we shall use '$\prec$'. This functor occurs in expressions of the type $\prec \langle \alpha_1, \alpha_2 \rangle$. A sentence built from '$\prec$' and its two name arguments is true *if and only if* exactly one of SeS_2-es occurs: II.1, II.3., II.9., II.10. Therefore, we can represent this functor by: '$\{1,3,9,10\}$'.Consequently, we define it as follows (according to instruction 3):

OF-Definition 22. $\prec \langle a,b\rangle \equiv = \langle a,b\rangle \vee \{3\} \langle a,b\rangle \vee \{9\} \langle a,b\rangle \vee \{10\} \langle a,b\rangle$ $\triangle$

Hence:

Lemma 5.8. *All OF_2-s are definable by means of 'ε' as the only OF.*

Proof. The set OF_2 is the union of SOF_2, POF_2, and $\{F_2\}$. All SOF_2-s are definable by means of 'ε' as the only OF (lemma 5.6). Functor 'F_2'

is definable by means of 'ε' as the only OF. All POF_2-s are definable by means of SOF_2 as the only OF-s (instruction 4). By the transitivity of definability, we obtain the demanded result. □

5.3. $OF_{k\geq 3}$-s

As we have said (definition 3.7) any $m \geq 3$-place SeS_m, say σ_m, of m different names, say $\langle \pi_1, \ldots, \pi_m \rangle$, is identical with the intersection of all SeS_2-es of all pairs being elements of $\{\pi_1, \ldots, \pi_m\} \times \{\pi_1, \ldots, \pi_m\}$. The deliberations hitherto led, suggest us that it is possible to define each m-place OF ($m \geq 3$) by means of OF_k-s ($2 \geq k$) as the only OF-s. How to execute such an operation?

If the number of SeS_m-es is settled, let it be k, the number of SOF_m is settled (since it is equal to the number of SeS_m-es -lemma 5.1 on p. 32) - it is k. If the number of SOF_m-s is settled, it is easy to determine the number of OF_m-s. It is the number of possible $\{0, 1\}$ variations of k-place sequence: 2^k. Hence:

Corollary 5.9. $\overline{\overline{OF_{m\geq 3}}} = 2^{\overline{\overline{SeS_{m\geq 3}}}}$

We could describe any $SeS_{m\geq 3}$ by describing all SeS_2-es of all ordered pairs being elements of $\{\pi_1, \ldots, \pi_m\} \times \{\pi_1, \ldots, \pi_m\}$. However, such a description would also contain some redundant information.

If we know the SeS_2 of π_1, π_2, we do not have to add any information regarding π_1, π_1, or π_2, π_2.The SeS_2 of π_i, π_i would inform us which of the states: II.1., II.9., or II.16. takes place between this name and itself. It is always some kind of identity. II.1., II.9 and II.16 differ only as to the question, whether I.1., I.2., or I.3. occurs. But such an information we have already got since we know the SeS_2 of π_1, π_2 , which determines not only the relation between π_1, π_2, but also SeS_1-es of π_1 and π_2. (do compare this statement with e.g. The Ontological Table). Next, if we know the SeS_2 of π_1, π_2, any additional information about the SeS_2 of π_2, π_1 is redundant.

Generally, for an m-place ($m \geq 3$) sequence of names $\langle \pi_1, \ldots, \pi_m \rangle$to determine their SeS_m is equivalent to determining the SeS_2-es of pairs: $\langle \pi_1, \pi_2 \rangle$, $\langle \pi_1, \pi_3 \rangle$, $\ldots$, $\langle \pi_1, \pi_m \rangle$, $\langle \pi_2, \pi_3 \rangle, \ldots, \langle \pi_2, \pi_m \rangle, \ldots, \langle \pi_{m-1}, \pi_m \rangle$.

There are $\frac{m^2-m}{2}$ such pairs. For each pair there are 16 possible SeS_2 that may take place for this pair. Therefore:

Corollary 5.10. *There are* $16^{\frac{m^2-m}{2}}$ *possible* $SeS_{m\geq 3}$*-es.*

Corollary 5.11. *There are* $2^{16^{\frac{m^2-m}{2}}}$ *possible* $OF_{m\geq 3}$*-s.*

Instruction 5. Let $m \geq 3$. We want to construct an expression univocally denoting an m-place SOF_m, say δ_m.

1. If δ_m is a SOF_m, than there is only one SeS_m of the sequence $\langle \pi_1, \dots, \pi_m \rangle$, (let it be σ_m), such that $\delta_m \langle \pi_1, \dots, \pi_m \rangle$ is true *if and only if* σ_m of $\langle \pi_1, \dots, \pi_m \rangle$ takes place.
2. We reduce σ_m of $\langle \pi_1, \dots, \pi_m \rangle$ to the intersection of SeS_2-es: $\langle \sigma_2^1, \dots, \sigma_2^{\frac{m^2-m}{2}} \rangle$ of (respectively) $\langle \pi_1, \pi_2 \rangle$, $\langle \pi_1, \pi_3 \rangle$, …, $\langle \pi_1, \pi_m \rangle$, $\langle \pi_2, \pi_3 \rangle$, …, $\langle \pi_2, \pi_m \rangle$, …, $\langle \pi_{m-1}, \pi_m \rangle$. Therefore we obtain the following sequence: $\langle \sigma_2^1, \dots, \sigma_2^{\frac{m^2-m}{2}} \rangle$
3. Each σ_2^i is one of 16 SeS_2-es. We construct k-place sequence $\langle s_1, \dots, s_k \rangle$ such that:
   (a) $16 \geq s_i \geq 1$
   (b) $s_i = j \equiv \sigma_2^i = II.j.$
4. To every number $16 \geq s_i \geq 1$ which may occur in the sequence such obtained, there is a formula corresponding to it. It is the formula built from SOF_2: '$\{s_i\}$' and its arguments $\langle a_x, b_y \rangle$, where x, y are the same indices which occur under the names in the pair corresponding to s_i.
5. Obtain the conjunction of all formulas $\tau_1, \dots, \tau_k$ corresponding to elements of $\langle s_1, \dots, s_k \rangle$. tel this conjunction be τ.
6. Construct the formula: $\delta_m \langle a_1, \dots, a_m \rangle$.
7. Construct the equivalence: $\delta_m \langle a_1, \dots, a_m \rangle \equiv \tau$. This is the definition looked after.

Hence, we have:

Lemma 5.12. *All* $SOF_{m\geq 3}$*-s are definable by means of '*ε*' as the only OF.*

Proof. We can define them by means of SOF_2-s, which are themselves definable by means of 'ε'. □

Now, we proceed to the last phase of our deliberations. It remains to give the procedure of representing and defining all $OF_{m\geq 3}$-s not being $SOF_{m\geq 3}$-s. The definitions of $falsum$ functors are trivial, so we omit them. What remains to be given, is a general instruction for referring to $POF_{m\geq 3}$-s, and an instruction for defining them.

Instruction 6. In order to represent any $POF_{m\geq 3}$, say $\delta_{m\geq 3}$, we have to:

1. Settle associated formulas (i.e. formulas being the result of applying a functor to its arguments, e.g. for δ_m it is $\delta_m\langle a_m, \ldots, a_m\rangle$) of all SOF_m -s in a sequence $\langle \tau_1, \tau_2, \ldots, \tau_k\rangle$ (Let k be the number of possible SOF_m-s). The order is non-essential (as far as we keep to the order once fixed).

2. When this order is fixed, for each POF_m δ_m there is exactly one sequence $\langle s_1, \ldots, s_k\rangle$ by which this POF_m may be represented, where each s_i ($k \geq i \geq 1$) is either 0, or 1, and $s_i = 1$ *if and only if* τ_i having value 1 is sufficient condition of $\delta_m\langle a_1, \ldots, a_m\rangle$ having value 1; otherwise it is 0. We can represent therefore any δ_m by such kind of a sequence.

3. It is possible to list exhaustively all POF_m-s by writing out all possible variations of k-place sequence of elements belonging to $\{0, 1\}$.

4. Such a notation may be farther abbreviated for practical purposes. Namely, instead of writing $\langle s_1, \ldots, s_k\rangle$, we can write $\langle s'_1, \ldots, s'_c\rangle_m$, where for every i, $k \geq s'_i \geq 1$, c is the number of all elements of $\langle s_1, \ldots, s_k\rangle$ being equal to 1, and for any s_u, u occurs in $\langle s'_1, \ldots, s'_c\rangle_m$ *if and only if* $s_u = 1$. The number m is equal to the number of arguments of the functor represented. However such a notation makes an exhaustive listing of possible functors and the description of defining procedure more complicated. Therefore, for purely theoretical purposes we still will use sequences of the kind: $\langle s_1, \ldots, s_k\rangle$.

5. This notation still can be abbreviated. The problem is, that if we stay on the level of what has been said in this instruction, all *verum*

functors will remain unabbreviated, and all functors represented by sequences in which almost all elements are 1 will not lose much of the longitude of the sequences representing them. That is why we can, for practical purposes, complicate our instruction. Namely, instead of writing $\langle s_1, \ldots, s_k \rangle$, we can write:

(a) If the number of elements of $\langle s_1, \ldots, s_k \rangle$ being equal to 1 is not bigger than $\frac{k}{2}$:
$\langle s'_1, \ldots, s'_c \rangle^1_m$, where c is the number of all elements of $\langle s_1, \ldots, s_k \rangle$ being equal to 1, and for any s_u, u occurs in $\langle s'_1, \ldots, s'_c \rangle$ *if and only if* $s_u = 1$. The number m is equal to the number of arguments of the functor represented.

(b) If the number of elements of $\langle s_1, \ldots, s_k \rangle$ being equal to 1 is bigger than $\frac{k}{2}$:
$\langle s'_1, \ldots, s'_c \rangle^0_m$, where c is the number of all elements of $\langle s'_1, \ldots, s'_c \rangle$ being equal to 0, and for any s_u , u occurs in $\langle s'_1, \ldots, s'_c \rangle$ *if and only if* $s_u = 0$. The number m is equal to the number of arguments of the functor represented.

We give a schema for defining POF_m-s:

Instruction 7. Assuming that we want to define an POF_m, let it be δ_m.

1. We have settled the sequence of associated formulas (i.e. formulas being the result of applying a functor to its arguments, e.g. for δ_m it is $\delta_m \langle a_m, \ldots, a_m \rangle$) of all SOF_m -s in a sequence $\langle \tau_1, \tau_2, \ldots, \tau_k \rangle$ (Let k be the number of possible SOF_m-s). We represented our δ_m by a sequence $\langle s_1, \ldots, s_k \rangle$, whereas each s_i $(k \geq i \geq 1)$ is either 0, or 1, and $s_i = 1$ *if and only if* τ_i having value 1 is sufficient condition of $\delta_m \langle a_1, \ldots, a_m \rangle$ having value 1; otherwise it is 0.

2. To any s_i there is a corresponding formula, namely τ_i. Form a disjunction of all formulas corresponding to those elements of the sequence $\langle s_1, \ldots, s_k \rangle$, which are equal to 1. Let the formula obtained be τ.

3. Form an associated formula of δ_m i.e. $\delta_m \langle a_1, \ldots, a_m \rangle$. Obtain the equivalence: $\delta_m \langle a_1, \ldots, a_m \rangle \equiv \tau$. This is the definition looked after.

Lemma 5.13. *All $OF_{m\geq 3}$-s are definable by means of 'ε' as the only ontological functor.*

Proof. All $SOF_{m\geq 3}$-s are definable by means of 'ε' as the only OF. All $POF_{m\geq 3}$-s are definable by means of $SOF_{m\geq 3}$-s, according to the procedure given above. Therefore, by extensionality for equivalence, and the fact, that definitions are equivalences, all $POF_{m\geq 3}$-s are definable by means of 'ε' as the only OF. [24] □

Next, we have:

Theorem 5.14. *All OF-s are definable by means of 'ε' as the only OF.*

Proof. All OF_1-s, all OF_2-s, and all $OF_{m\geq 3}$-s are definable by means of 'ε' as the only OF. Therefore, all OF-s are definable by means of 'ε' as the only OF. □

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[24] Obviously, we might have as well treated definitions as formulated in meta-language; the consequence would still hold, by definitional extensionality.