REPORTS ON MATHEMATICAL LOGIC
37 (2003), 113–123

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AXIOMATIC EXTENSIONS OF THE NILPOTENT MINIMUM LOGIC

A b s t r a c t. In this paper we characterize, classify and axiomatize all axiomatic extensions of the Nilpotent Minimum Logic. Every axiomatic extension is complete with respect to a class of NM-chains. Given a family of NM-chains the number of elements of the largest odd finite NM-chain in the family and the number of elements of the largest even finite NM-chain in the family turns out to be a complete classifier.

Preliminaries

The nilpotent minimum logic, NML for short, was firstly introduced by F. Esteva and L. Godo in [2] in order to formalize the logic of the nilpotent minimum t-norms. J. Fodor in [3] defined the nilpotent minimum t-norms as examples of some involutive left continuous t-norms which are not continuous (and hence they are not Łukasiewicz

Received 18 November 2002

This work is partially supported by Grant 2001SGR-0017 of D.G.R. of Generalitat de Catalunya, by Grant BFM2001-3329 of D.G.I.C.Y.T. of Spain and by Spanish CICYT project LOGFAC (TIC2001-1577-C03-01)

t-norms). NML is the logic obtained from the monoidal t-norm logic, MTL for short, defined by Esteva and Godo in [2], by adding the involutive condition $\neg\neg\varphi \rightarrow \varphi$ and the nilpotent minimum condition $(\psi * \varphi \rightarrow \bot) \vee (\psi \wedge \varphi \rightarrow \psi * \varphi)$. Hence the calculus is given by the following axioms.

A1 $(\varphi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\varphi \rightarrow \chi))$

A2 $(\varphi * \psi) \rightarrow \varphi$

A3 $(\varphi * \psi) \rightarrow (\psi * \varphi)$

A4 $(\varphi \wedge \psi) \rightarrow \varphi$

A5 $(\varphi \wedge \psi) \rightarrow (\psi \wedge \varphi)$

A6 $(\varphi * (\varphi \rightarrow \psi)) \rightarrow (\varphi \wedge \psi)$

A7a $(\varphi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\varphi * \psi) \rightarrow \chi)$

A7b $((\varphi * \psi) \rightarrow \chi) \rightarrow (\varphi \rightarrow (\psi \rightarrow \chi))$

A8 $((\varphi \rightarrow \psi) \rightarrow \chi) \rightarrow (((\psi \rightarrow \varphi) \rightarrow \chi) \rightarrow \chi)$

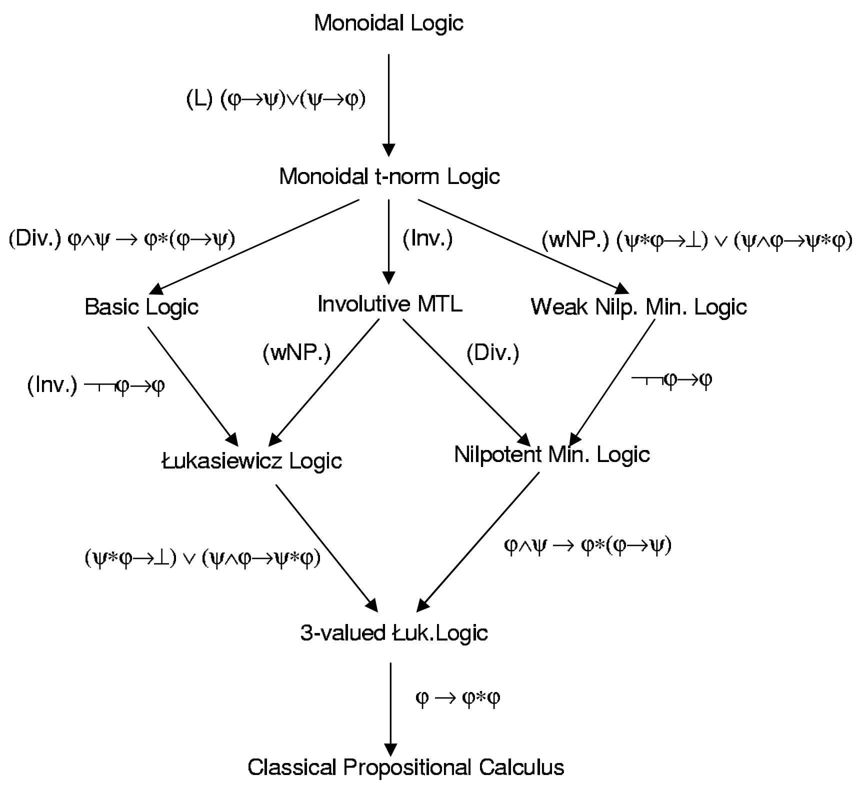
A9 $\bot \rightarrow \varphi$

A10 $\neg\neg\varphi \rightarrow \varphi$

A11 $(\psi * \varphi \rightarrow \bot) \vee (\psi \wedge \varphi \rightarrow \psi * \varphi)$

and the rule of *Modus Ponens* $\langle\{\varphi \rightarrow \psi, \varphi\}, \psi\rangle$. Notice that NML is an axiomatic extension of IMTL and therefore of MTL and ML (see for instance [4, 2, 6]) . In [2] it is proved that NML is complete with respect to the class of NM-algebras, moreover it is proved to be standard complete (complete with respect to the standard NM-algebra, i.e the NM-algebra whose universe is [0,1] cf. Example 1). In the next figure we include the diagram of some of the mentioned logics plus some well-known logics and their corresponding axioms.

Fig1.



A nilpotent minimum algebra, NM-algebra for short, is a bounded residuated lattice $\mathbf{A} = \langle A, *, \rightarrow, \wedge, \vee, \neg, 0, 1\rangle$, where $\wedge$ and $\vee$ are meet and join, $\langle *, \rightarrow\rangle$ is a residuated pair, $\neg$ is the associated negation (i.e. $\neg x =_{def} x \rightarrow 0$) and 0 is the lower bound and 1 is the upper bound, and moreover it satisfies the following conditions:

Pre-linearity equation (L) $(x \rightarrow y) \vee (y \rightarrow x) \approx 1$

Involutive equation (I) $\neg\neg x \approx x$

Nilpotent minimum equation (WNM) $(x*y \rightarrow 0) \vee (x \wedge y \rightarrow x*y) \approx 1$

Since the class of all bounded residuated lattices, also called integral commutative l-monoids [4], is equational (see [5]) and the three conditions of (L), (I) and (WNM) are equational, then the class of all NM-algebras, denoted by NM, is a variety. In fact NM is a proper subvariety of the varieties ML, MTL, IMTL and WNM-algebras, moreover NM=IMTL∩WNM.

We say that a NM-algebra is a NM-chain, provided that it is totally ordered. Since the class of NM-algebras is a subvariety of MTL-algebras, the decomposition property is also valid (see [2, Proposition 3]).

Proposition 1. *Each NM-algebra is representable as a subdirect product of NM-chains.* □

From (WNM) it is easy to see that given a totally ordered set A with upper bound 1 and lower bound 0 equipped with an involutive negation $\neg$ dually order preserving, if we define $\wedge$, $\vee$ as meet and join and for every $a, b \in A$,

$$a * b = \begin{cases} 0, & \text{if } b \leq \neg a; \\ a \wedge b, & \text{otherwise.} \end{cases} \qquad a \to b = \begin{cases} 1, & \text{if } a \leq b; \\ \neg a \vee b, & \text{otherwise.} \end{cases},$$

then $\mathbf{A} = \langle A, *, \to, \wedge, \vee, \neg, 0, 1\rangle$ is a NM-chain. Moreover every NM-chain is of this form.

Given an NM-algebra $\mathbf{A}$ and given an element $a \in A$, we say that a is a *positive element of* $\mathbf{A}$ provided that $a > \neg a$. We say that $a \in A$ is a *negative element of* $\mathbf{A}$ provided that $a < \neg a$. We say that a is a *negation fixpont* iff $a = \neg a$. Höhle proves in [4] that in any ML-algebra (in particular, in any NM-algebra) the negation fixpoint, if it exists, is unique. It follows from the construction given above that any NM-chain is determined and generated by its positive elements and its negation fixpoint (if it exists).

The following examples of NM-algebras play an important role in this paper.

Example 1. $[\mathbf{0}, \mathbf{1}] = \langle [0, 1], *, \to, \wedge, \vee, \neg, 0, 1\rangle$
where $\wedge$ and $\vee$ are meet and join with the usual order,
$\neg x = 1 - x$,
$$x * y = \begin{cases} 0, & \text{if } y \leq 1 - x; \\ x \wedge y, & \text{otherwise.} \end{cases}$$
$$x \to y = \begin{cases} 1, & \text{if } x \leq y; \\ \neg x \vee y, & \text{otherwise.} \end{cases}$$
Notice that $[0, 1/2)$ are the positive elements of $[\mathbf{0}, \mathbf{1}]$ and $1/2$ is its negation fixpont.

Example 2. For every natural number n,
$\mathbf{A_{2n+1}} = \langle [-n, n] \cap \mathbb{Z} = \{-n, \ldots, 0, \ldots, n\}, *, \to, \wedge, \vee, \neg, -n, n\rangle$
where $\wedge$ and $\vee$ are meet and join with the usual order, negation also defined as usual $\neg x = -x$ and for every natural numbers $0 < m, k \leq n$,

$m * k = \min\{m, k\}, (-m) * (-k) = -n, m * 0 = 0, \quad -m * 0 = 0 * 0 = -n,$

$$m * (-k) = \begin{cases} -n, & \text{if } m \leq k; \\ -k, & \text{otherwise.} \end{cases}$$

$$m \to k = \begin{cases} n, & \text{if } m \leq k; \\ k, & \text{otherwise.} \end{cases}, \qquad -m \to -k = \begin{cases} n, & \text{if } k \leq m; \\ m, & \text{otherwise.} \end{cases},$$

$m \to -k = \max\{-m, -k\}, \qquad -m \to k = n, \qquad m \to 0 = 0 \to -m = 0,$

$-m \to 0 = 0 \to m = 0 \to 0 = n.$

Observe that $\mathbf{A_1}$ is the trivial algebra and $\mathbf{A_3}$ is polynomially equivalent to the 3-element MV-algebra $Ł_3$.

Example 3. For every natural number $n > 0$,
$\mathbf{A_{2n}} = \langle [-n, n] \cap \mathbb{Z} \setminus \{0\}, *, \to, \wedge, \vee, \neg, -n, n \rangle$
The subalgebra of $\mathbf{A_{2n+1}}$ obtained by removing the negation fixpoint 0 from the universe of $\mathbf{A_{2n+1}}$.
Notice that $\mathbf{A_2}$ is polynomially equivalent to the 2-element Boolean algebra.

In fact, given a NM-chain $\mathbf{C}$ with negation fixpoint $c \in C$, $C \setminus \{c\}$ is always the universe of a subalgebra of $\mathbf{C}$, which we denote by $\mathbf{C}^-$.

Example 4. $[\mathbf{0}, \mathbf{1}]^- = \langle [0, 1] \setminus \{1/2\}, *, \to, \wedge, \vee, \neg, 0, 1 \rangle$
the subalgebra of $[\mathbf{0}, \mathbf{1}]$ whose universe is $[0, 1] \setminus \{1/2\}$.

The following relations among NM-algebras follow from their definitions.

Proposition 2.

$\mathbf{A_{2m+1}}$ *is a subalgebra of* $\mathbf{A_{2n+1}}$ *for every* $m \leq n$,

$\mathbf{A_{2m}}$ *is a subalgebra of* $\mathbf{A_{2n+1}}$ *for every* $0 < m \leq n$,

$\mathbf{A_{2m}}$ *is a subalgebra of* $\mathbf{A_{2n}}$ *for every* $0 < m \leq n$,

$\mathbf{A_m}$ *is a embeddable into* $[\mathbf{0}, \mathbf{1}]$ *for every* $m > 1$,

$\mathbf{A_{2m}}$ *is a embeddable into* $[\mathbf{0}, \mathbf{1}]^-$ *for every* $m > 0$.

It follows from [7] and it is proved in [2] that up to isomorphism there is only one nilpotent minimum algebra defined on [0,1], the one defined in Example 1 usually called the *standard NM-algebra.* Moreover it is easy to see that $\mathbf{A_{2n}}$ and $\mathbf{A_{2n+1}}$ are, up to isomorphism, the only nilpotent minimum chains with exactly 2n elements and 2n+1 elements respectively. Notice moreover that any finitely generated subalgebra of a nontrivial NM-chain is finite and therefore isomorphic to $\mathbf{A_{2n}}$ or to $\mathbf{A_{2n+1}}$ for some $n > 0$.

Main results

Our purpose in this paper is to characterize, classify and axiomatize all axiomatic extensions of NML or equivalently all proper subvarieties of NM. Next results follow from the NML completeness proof given in [2].

Theorem 1. *A variety of NM-algebras is a proper subvariety of NM if and only if it does not contain some* $\mathbf{A_k}$ *with* $1 < k$.

Proof. The *if* part is trivial. Let us assume that V is a proper variety, hence there is an equation $\varepsilon \approx \delta$ with m variables, say $\{x_0, \ldots, x_{m-1}\}$, satisfied in V and not satisfied in NM. Hence there are a non trivial NM-algebra $\mathbf{A} \notin V$ and some elements $a_0, \ldots, a_{m-1} \in A$ such that $\varepsilon(a_0, \ldots, a_{m-1}) \neq \delta(a_0, \ldots, a_{m-1})$. Since any NM-algebra decomposes into a subdirect product of NM-chains (see Proposition 1), we may assume without loss of generality that $\mathbf{A}$ is a NM-chain. Now, let $\mathbf{B}$ be the subalgebra of $\mathbf{A}$ generated by $\{a_0, \ldots, a_{m-1}\}$. Since $\mathbf{B}$ is finitely generated $\mathbf{B} \cong \mathbf{A_k}$ for some $1 < k$, and $\mathbf{A_k} \notin V$, because $\mathbf{A_k}$ does not satisfy $\varepsilon \approx \delta$. □

Corollary 1. $NM = \mathbb{V}(\{\mathbf{A_n} \ : \ n \in \omega, \ 0 < n\}) = \mathbb{V}(\{\mathbf{A_{2n+1}}, n \in \omega\})$.

Corollary 2. *Let* $\mathbf{A}$ *be an infinite NM-chain containing the negation fixpoint. Then* $\mathbb{V}(\mathbf{A}) = NM$.

Since any finitely generated chain is finite, by Proposition 1 and using a similar argument as in the proof of Theorem 1 it is easy to see that:

Proposition 3. *Every variety of NM-algebras is generated by its finite NM-chains.*

The next proposition follows from the above results and completeness for NML

Proposition 4. *NML is decidable.*

Definition 1. For each $n > 0$ we consider the following formulas:

$$\text{A12}_n \quad S_n(x_0, \ldots, x_n) = \bigwedge_{i<n} ((x_i \to x_{i+1}) \to x_{i+1}) \to \bigvee_{i<n+1} x_i$$

$$\text{A13} \quad BP(x) = \neg(\neg x^2)^2 \leftrightarrow (\neg(\neg x)^2)^2$$

Theorem 2. *1. A NM-chain satisfies* $S_n(x_0, \ldots, x_n) \approx 1$ *if and only if it has less than* $2n + 2$ *elements.*

2. *A nontrivial NM-chain satisfies* $BP(x) \approx 1$ *if and only if it does not contain the negation fixpoint.*

Proof.

1. Assume that $\mathbf{A}$ is a NM-chain with less than $2n + 2$ elements. Let $a_0, \ldots, a_n \in A$. Note that if $a_i \leq a_{i+1}$ then $(a_i \rightarrow a_{i+1}) \rightarrow a_{i+1}) = a_{i+1}$, therefore $\bigwedge_{i<n}((a_i \rightarrow a_{i+1}) \rightarrow a_{i+1}) \leq \bigvee_{i<n+1} a_i$. Assume then, that $a_i > a_{i+1}$ for every $i < n$ and $a_0 \neq 1$ (If $a_0 = 1$, then $\mathrm{S}_n(a_0, \ldots, a_n) = 1$). Since A has at most 2n+1 elements there are some negative elements a_i and a_{i+1}. In that case $(a_i \rightarrow a_{i+1}) \rightarrow a_{i+1}) = a_i$, therefore $\bigwedge_{i<n}((a_i \rightarrow a_{i+1}) \rightarrow a_{i+1}) \leq \bigvee_{i<n+1} a_i$ and $\mathrm{S}_n(a_0, \ldots, a_n) = 1$.
   To prove the converse, let $\mathbf{A}$ be a NM-chain with at least 2n+2 elements. Hence there are $a_0, \ldots, a_n \in A$ such that $a_0 \neq 1$, $a_i > a_{i+1} \geq \neg a_i$ for every $i < n$. Then for every $i < n$, $(a_i \rightarrow a_{i+1}) \rightarrow a_{i+1}) = 1$, while $\bigvee_{i<n+1} a_i = a_0 \neq 1$. Thus $\mathbf{A}$ does not satisfy $S_n(x_0, \ldots, x_n) \approx 1$.

2. If $\mathbf{A}$ is a NM-chain such that it contains an element $a = \neg a$, then notice that $(\neg(\neg a)^2)^2 = (\neg a^2)^2 = (\neg 0)^2 = 1$ while $\neg(\neg a^2)^2 = \neg(\neg 0)^2 = 0$. Hence it does not satisfy $BP(X) \approx 1$. On the other hand, assume that for every element $a \in A$ $a \neq \neg a$. In that case it is easy to see that $(\neg(\neg a)^2)^2 = \neg(\neg a^2)^2 = 0$ if a is negative and $(\neg(\neg a)^2)^2 = \neg(\neg a^2)^2 = 1$ if a is positive.

□

Theorem 3. *Every proper nontrivial subvariety of NM is of one of the following types:*

1. $\mathbb{V}([\mathbf{0},\mathbf{1}]^-) = \mathbb{V}(\{\mathbf{A_{2k}} : k \in \omega\})$
2. $\mathbb{V}(\mathbf{A_{2m+1}})$ *for some* $m > 0$
3. $\mathbb{V}(\mathbf{A_{2n}})$ *for some* $n > 0$
4. $\mathbb{V}([\mathbf{0},\mathbf{1}]^-, \mathbf{A_{2m+1}})$ *for some* $m > 0$
5. $\mathbb{V}(\mathbf{A_{2n}}, \mathbf{A_{2m+1}})$ *for some* $m, n \in \omega$ *such that* $0 < m < n$

Moreover, if Σ is any set of equations axiomatizing NM, then

1. $\mathbb{V}([\mathbf{0},\mathbf{1}]^-)$ *is axiomatized by Σ plus the equation* $BP(x) \approx 1$,
2. $\mathbb{V}(\mathbf{A_{2m+1}})$ *is axiomatized by Σ plus the equation* $S_m(x_0, \ldots, x_m) \approx 1$,
3. $\mathbb{V}(\mathbf{A_{2n}})$ *is axiomatized by Σ plus the equations* $S_n(x_0, \ldots, x_n) \approx 1$ *and* $BP(x) \approx 1$,
4. $\mathbb{V}([\mathbf{0},\mathbf{1}]^-, \mathbf{A_{2m+1}})$ *is axiomatized by Σ plus the equation* $BP(x) \vee S_m(x_0, \ldots, x_m) \approx 1$,
5. $\mathbb{V}(\mathbf{A_{2n}}, \mathbf{A_{2m+1}})$ *with $m < n$ is axiomatized by Σ plus the equation* $(BP(x) \wedge S_n(x_0, \ldots, x_n)) \vee S_m(x_0, \ldots, x_m) \approx 1$.

Proof. After Theorem 2, it is easy to see that the standard algebra does not satisfy any of the new equations proposed to axiomatize each variety. Moreover every variety of type 1. to 5. satisfies its proposed equations. Consequently all varieties of type 1. to 5. are proper subvarieties of NM.

Assume that V is a proper subvariety of NM. Since any NM variety is generated by its finite chains (cf. Proposition 3), we will only deal with finite chains. Let $m = \max\{k \in \omega \ : \ \mathbf{A_{2k+1}} \in V\}$ (this maximum exists by Theorem 1). Let $\gamma = \sup\{k \in \omega \ : \ \mathbf{A_{2k}} \in V\}$, notice that in this case γ may be ω, but in every case $m \leq \gamma$, because $\mathbf{A_{2m}}$ is a subalgebra of $\mathbf{A_{2m+1}} \in V$. We proceed now by cases:

($m = 0$**,** $\gamma = \omega$) This means that there is no NM-chain in V containing the negation fixpoint, except the trivial algebra, therefore all nontrivial finite NM-chains are of the form $\mathbf{A_{2k}}$. Moreover, since $\gamma = \omega$, for every $k \in \omega$, $\mathbf{A_{2k}} \in V$. Consequently, $V = \mathbb{V}(\{\mathbf{A_{2k}} : k \in \omega\})$. The equality $\mathbb{V}([\mathbf{0},\mathbf{1}]^-) = \mathbb{V}(\{\mathbf{A_{2k}} : k \in \omega\})$ follows from the fact that $\{\mathbf{A_{2k}} : k \in \omega\}$ is up to isomorphism the family of all finitely generated subalgebras of $[\mathbf{0},\mathbf{1}]^-$.

($m \neq 0$**,** $\gamma = m$) In this case every nontrivial finite NM-chain in V is a subalgebra of $\mathbf{A_{2m+1}} \in V$, therefore $V = \mathbb{V}(\mathbf{A_{2m+1}})$.

($m = 0$**,** $\gamma = n$) As in first case, there is no NM-chain in V containing the negation fixpoint, except the trivial algebra, therefore all nontrivial finite NM-chains are of the form $\mathbf{A_{2k}}$. Moreover since $\gamma = n$, every nontrivial finite NM-chain in V is a subalgebra of $\mathbf{A_{2n}} \in V$, therefore $V = \mathbb{V}(\mathbf{A_{2n}})$.

($m \neq 0$**,** $\gamma = \omega$) In this case the only nontrivial finite NM-chains in V containing the negation fixpoint are all subalgebras of $\mathbf{A}_{2m+1} \in V$. Moreover since $\gamma = \omega$, for every $k \in \omega$, $\mathbf{A_{2k}} \in V$, therefore $V = \mathbb{V}(\{\mathbf{A_{2k}} : k \in \omega\} \cup \{\mathbf{A_{2m+1}}\}) = \mathbb{V}([\mathbf{0}, \mathbf{1}]^-, \mathbf{A_{2m+1}})$.

($m \neq 0$**,** $\gamma = n > m$) As in the preceding case the only nontrivial finite NM-chains in V containing the negation fixpoint are all subalgebras of $\mathbf{A_{2m+1}} \in V$. Moreover since $\gamma = n$, every nontrivial finite NM-chain in V without negation fixpoint is a subalgebra of $\mathbf{A_{2n}} \in V$. Hence $V = \mathbb{V}(\mathbf{A_{2n}}, \mathbf{A_{2m+1}})$.

Finally, the axiomatization of every subvariety of NM follows from Theorem 2 and the fact that any NM-chain satisfies:

$$x \wedge y \approx 1 \text{ iff } x \approx 1 \text{ and } y \approx 1 \qquad \text{and} \qquad x \vee y \approx 1 \text{ iff } x \approx 1 \text{ or } y \approx 1.$$

□

Working with the relations among NM-algebras established in Proposition 2 it is easy to obtain the following relations among NM-varieties:

$\mathbb{V}(\mathbf{A_{2n+1}}) \subseteq \mathbb{V}(\mathbf{A_{2m+1}})$ for every $n \leq m$

$\mathbb{V}(\mathbf{A_{2n}}) \subseteq \mathbb{V}(\mathbf{A_{2m+1}})$ for every $0 < n \leq m$

$\mathbb{V}(\mathbf{A_{2n}}) \subseteq \mathbb{V}(\mathbf{A_{2m}})$ for every $0 < n \leq m$

$\mathbb{V}(\mathbf{A_{2n}}) \subseteq \mathbb{V}([\mathbf{0}, \mathbf{1}]^-)$ for every $n > 0$

$\mathbb{V}([\mathbf{0}, \mathbf{1}]^-) \cap \mathbb{V}(\mathbf{A_{2m+1}}) = \mathbb{V}(\mathbf{A_{2m}})$ for every $m > 0$

$\mathbb{V}(\mathbf{A_{2n}}) \cap \mathbb{V}(\mathbf{A_{2m+1}}) = \mathbb{V}(\mathbf{A_{2\min\{n,m\}}})$ for every $n, m > 0$

It follows from the completeness theorem for NML that there is a lattice isomorphism from the lattice of all subvarieties of NM into the lattice of all axiomatic extensions of NML (In fact NML is algebraizable in the sense of Blok and Pigozzi [1], therefore we have a lattice isomorphism from the lattice of all subquasivarieties of NM into the lattice of all finitary extensions of NML).

Theorem 4. *All proper consistent axiomatic extensions of* NML *are: For every natural numbers* $n, m > 0$

1. NM^- = NML *plus A13*

2. NM2m+1 = NML *plus* $A12_m$

3. NM2n = NML *plus* $A12_n$ *and A13*

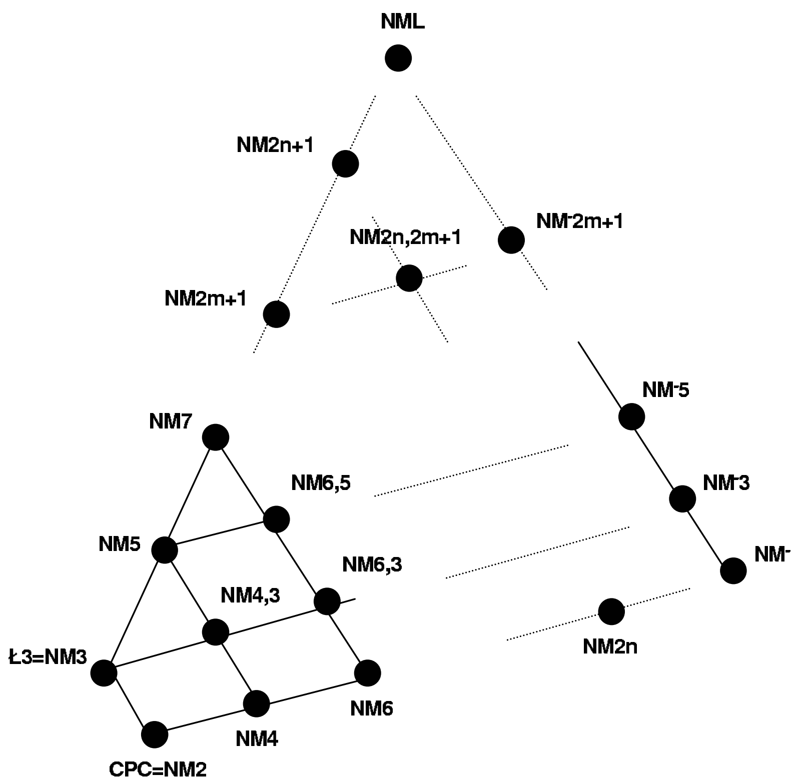
4. NM^-2m+1 = NML *plus* $A13 \vee A12_m$

5. NM2n,2m+1 = NML *plus* $(A12_n \wedge A13) \vee A12_m$ *with* $n > m$

Corollary 3. *Every proper axiomatic extension of* NML *is decidable.*

Finally translating the relations among NM varieties stated above to relations among axiomatic extensions of NML, in the next figure we sketch the lattice of axiomatic extensions of NML.

Fig2. Lattice of Axiomatic Extensions of NML



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