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COUNTING QUASIVARIETIES OF EQUIVALENTIAL ALGEBRAS

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The purely equivalential fragment of the intuitionistic logic was characterized from an algebraic point of view by J.K.Kabziński and A.Wroński [2]. They introduced the variety of equivalential algebras and showed that it consists of $(\leftrightarrow)$–subreducts of Heyting algebras, where

$$x \leftrightarrow y = (x \to y) \wedge (y \to x).$$

Recently equivalential algebras have been studied by K.Słomczyńska [3]. She introduced a representation for finite equivalential algebras, which proved to be extremely useful in studying their structure. With the aid of this tool she proved that each quasivariety of equivalential algebras is actually a variety. This together with an earlier result of A. Wroński [4]

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gives that for every finite equivalential algebra $\underline{\mathrm{A}}$ the lattice $L(\mathrm{Q}(\underline{\mathrm{A}}))$, of all subquasivarieties of the quasivariety $\mathrm{Q}(\underline{\mathrm{A}})$ generated by $\underline{\mathrm{A}}$, is finite.

Although constants do not change congruences they do change subalgebras. Thus some of the properties that depend on subalgebras can change when adding constants.

We demonstrate that adding constants can destroy many of the nice properties equivalential algebras do enjoy. In particular we show that none of the following remains true after expanding equivalential algebras by a constant:

- every quasivariety is a variety
- every finitely generated quasivariety has finitely many subquasivarieties
- every finitely generated quasivariety is finitely axiomatizable.

Generally we prove the following (cf. [1]):

Theorem 1. *For the variety $\mathcal{V}$ generated by a finite equivalential algebra expanded by a single constant c the following conditions are equivalent:*

(1) *$\mathcal{V}$ has finitely many subquasivarieties,*

(2) *every subquasivariety of $\mathcal{V}$ is finitely axiomatizable,*

(3) *the constant c is regular (i.e., $\mathcal{V} \models (c \leftrightarrow x) \leftrightarrow x = c$).*

References

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[2] Kabziński J.K., Wroński A., On equivalential algebras, *Proceedings of the 1975 International Symposium on Multiple-Valued Logic*, Indiana University, Bloomington, May 13–16, 1975, pp.419–428.

[3] Słomczyńska K., *Equivalential algebras*, Ph.D. Thesis, Jagiellonian University, Kraków 1994.

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